Data Based Design of 3 Term Controllers

History

- Classical Control - single controller (PID, lead/lag) is designed based on transfer function or frequency response data

History

- Classical Control - single controller (PID, lead/lag) is designed based on transfer function or frequency response data
- Modern/Post Modern Control (H_∞, H_2, ℓ_1) - optimal controller of high order based on state space model

History

- Classical Control - single controller (PID, lead/lag) is designed based on transfer function or frequency response data
- Modern/Post Modern Control (H_∞, H_2, ℓ_1) - optimal controller of high order based on state space model
- Fuzzy Neural Control- model free approach but no guarantee of stability or performance

History

- Classical Control - single controller (PID, lead/lag) is designed based on transfer function or frequency response data
- Modern/Post Modern Control (H_∞, H_2, ℓ_1) - optimal controller of high order based on state space model
- Fuzzy Neural Control- model free approach but no guarantee of stability or performance
- Our Recent Results - complete set of PID controllers achieving stability and performance based on transfer function model

History

- Classical Control - single controller (PID, lead/lag) is designed based on transfer function or frequency response data
- Modern/Post Modern Control (H_∞, H_2, ℓ_1) - optimal controller of high order based on state space model
- Fuzzy Neural Control- model free approach but no guarantee of stability or performance
- Our Recent Results - complete set of PID controllers achieving stability and performance based on transfer function model
- Present Paper - extend these results to the case where only data is available

Drawbacks of Identification

- States have no physical significance

Drawbacks of Identification

- States have no physical significance
- Parameters in the model have no physical significance

Drawbacks of Identification

- States have no physical significance
- Parameters in the model have no physical significance
- State variables have no meaningful dimensions or units

Main Idea:

- To synthesize the set of all stabilizing PID/First Order controllers from experimental data of the plant rather than from mathematical models.

Main Idea:

- To synthesize the set of all stabilizing PID/First Order controllers from experimental data of the plant rather than from mathematical models.
- To synthesize the set of all PID/First Order controllers satisfying the given performance requirements from the experimental data of the plant.

Main Idea:

- To synthesize the set of all stabilizing PID/First Order controllers from experimental data of the plant rather than from mathematical models.

- To synthesize the set of all PID/First Order controllers satisfying the given performance requirements from the experimental data of the plant.

  - Knowledge of the frequency response magnitude and phase, equivalently,

$$P(j\omega), \quad \omega \in [0, \infty)$$

Main Idea:

- To synthesize the set of all stabilizing PID/First Order controllers from experimental data of the plant rather than from mathematical models.
- To synthesize the set of all PID/First Order controllers satisfying the given performance requirements from the experimental data of the plant.
  - Knowledge of the frequency response magnitude and phase, equivalently,

$$P(j\omega), \quad \omega \in [0, \infty)$$

  - Knowledge of the number of RHP poles p^+.

Assumption

- The plant is stabilizable (i.e., $N(s)$ and $D(s)$ are coprime). where

$$P(s) = \frac{N(s)}{D(s)}$$

Assumption

- The plant is stabilizable (i.e., $N(s)$ and $D(s)$ are coprime). where

$$P(s) = \frac{N(s)}{D(s)}$$

- The plant has no $j\omega$ poles.

Assumption

- The plant is stabilizable (i.e., $N(s)$ and $D(s)$ are coprime). where

$$P(s) = \frac{N(s)}{D(s)}$$

- The plant has no $j\omega$ poles.
- Only available information

Assumption

- The plant is stabilizable (i.e., $N(s)$ and $D(s)$ are coprime). where

$$P(s) = \frac{N(s)}{D(s)}$$

- The plant has no $j\omega$ poles.

- Only available information
  - $P(j\omega)$ for $\omega \in [0, \infty)$

Assumption

- The plant is stabilizable (i.e., $N(s)$ and $D(s)$ are coprime). where

$$P(s) = \frac{N(s)}{D(s)}$$

- The plant has no $j\omega$ poles.
- Only available information
  - $P(j\omega)$ for $\omega \in [0, \infty)$
  - Number of plant RHP poles, p^+.

Preliminaries

Consider a real rational function

$$R(s) = \frac{A(s)}{B(s)}$$

where $A(s)$ and $B(s)$ are polynomials of real coefficients with degrees m and n, respectively. Assume that $A(s)$ and $B(s)$ have no zeros on the $j\omega$ axis.

Preliminaries

Consider a real rational function

$$R(s) = \frac{A(s)}{B(s)}$$

where $A(s)$ and $B(s)$ are polynomials of real coefficients with degrees m and n, respectively. Assume that $A(s)$ and $B(s)$ have no zeros on the $j\omega$ axis.

Write

$$R(j\omega) = R_r(\omega) + jR_i(\omega)$$

where $R_r(\omega)$ and $R_i(\omega)$ are real rational functions in ω. Note that $R_r(\omega)$ and $R_i(\omega)$ have no real poles for $\omega \in (-\infty, +\infty)$.

Preliminaries

Consider a real rational function

$$R(s) = \frac{A(s)}{B(s)}$$

where $A(s)$ and $B(s)$ are polynomials of real coefficients with degrees m and n, respectively. Assume that $A(s)$ and $B(s)$ have no zeros on the $j\omega$ axis.

Write

$$R(j\omega) = R_r(\omega) + jR_i(\omega)$$

where $R_r(\omega)$ and $R_i(\omega)$ are real rational functions in ω. Note that $R_r(\omega)$ and $R_i(\omega)$ have no real poles for $\omega \in (-\infty, +\infty)$.

Let

$$0 = \omega_0 < \omega_1 < \omega_2 < \cdots < \omega_{l-1}$$

and define $\omega_l = \infty^-$ denote the finite zeros of $R_i(\omega) = 0$

Real Hurwitz Signature Lemma

Define

$$\text{sgn}[x] = \begin{cases} +1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

Real Hurwitz Signature Lemma

Define

$$\text{sgn}[x] = \begin{cases} +1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

If $n - m$ is even

$$\sigma(R) = \left(\text{sgn}[R_r(\omega_0^+)] + 2\sum_{j=1}^{l-1} (-1)^j \text{sgn}[R_r(\omega_j)] + (-1)^l \text{sgn}[R_r(\omega_l)] \right)$$
$$\cdot (-1)^{l-1} \text{sgn}[R_i(\infty^-)]$$

If $n - m$ is odd

$$\sigma(R) = \left(\text{sgn}[R_r(\omega_0)] + 2\sum_{j=1}^{l-1} (-1)^j \text{sgn}[R_r(\omega_j)] \right) (-1)^{l-1} \text{sgn}[R_i(\infty^-)]$$

Complex Hurwitz Signature Lemma

Consider a complex rational function

$$Q(s) = \frac{D(s)}{E(s)}, \qquad Q(j\omega) = Q_r(\omega) + jQ_i(\omega)$$

where $Q_r(\omega)$ and $Q_i(\omega)$ are real rational functions. $Q_r(\omega)$ and $Q_i(\omega)$ have no real poles for $\omega \in (-\infty, +\infty)$.

Complex Hurwitz Signature Lemma

Consider a complex rational function

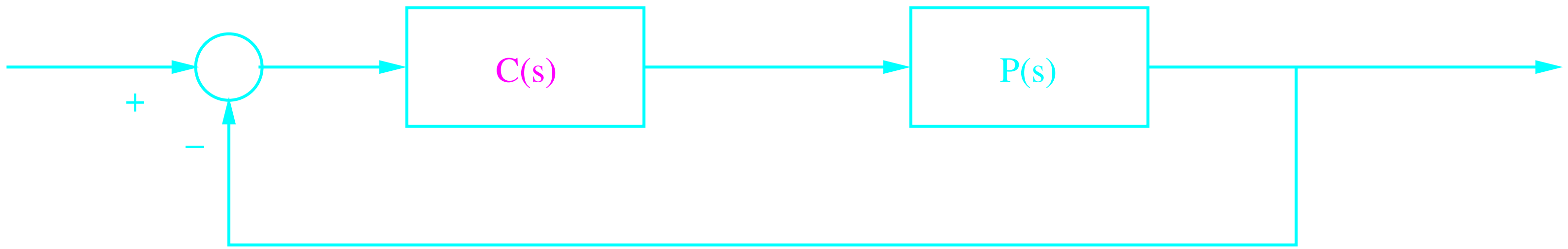
$$Q(s) = \frac{D(s)}{E(s)}, \qquad Q(j\omega) = Q_r(\omega) + jQ_i(\omega)$$

where $Q_r(\omega)$ and $Q_i(\omega)$ are real rational functions. $Q_r(\omega)$ and $Q_i(\omega)$ have no real poles for $\omega \in (-\infty, +\infty)$.

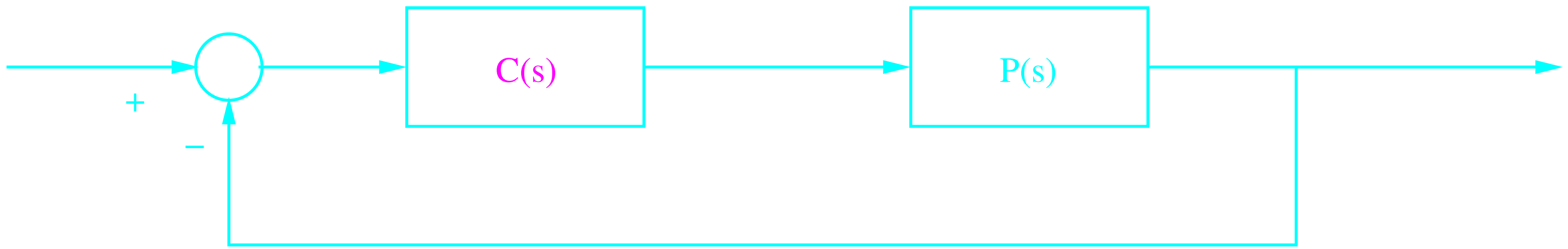
Lemma

$$\sigma(Q) = \left(\sum_{j=1}^{l-1}(-1)^{l-1-j}\operatorname{sgn}[Q_r(\omega_j)]\right)\operatorname{sgn}[Q_i(\infty^-)]$$

PID controller design



PID controller design



Let the PID controller be of the form

$$C(s) = \frac{K_i + K_p s + K_d s^2}{s(1+sT)}, \qquad T > 0$$

Main Results

- The complete set of stabilizing PID gains for a given LTI plant can be found from the frequency response data $P(j\omega)$ and the knowledge of the number of RHP poles.

Main Results

- The complete set of stabilizing PID gains for a given LTI plant can be found from the frequency response data $P(j\omega)$ and the knowledge of the number of RHP poles.

- Using the result above, the subset of the PID gains that satisfy the several given performance requirements.

PID Controller Design

Let us consider the plant and PID controller pair of the form:

$$
\begin{aligned}
P(s) &= \frac{N(s)}{D(s)} \\
C(s) &= \frac{K_i + K_p s + K_d s^2}{s(1+sT)}, \quad T > 0
\end{aligned}
$$

where $\deg[D(s)] = n$ and $\deg[N(s)] = m$.

PID Controller Design

Let us consider the plant and PID controller pair of the form:

$$P(s) = \frac{N(s)}{D(s)}$$

$$C(s) = \frac{K_i + K_p s + K_d s^2}{s(1+sT)}, \quad T > 0$$

where $\deg[D(s)] = n$ and $\deg[N(s)] = m$.

Then consider the rational function

$$F(s) = s(1+sT) + \left(K_i + K_p s + K_d s^2\right) P(s)$$

PID Controller Design

Let us consider the plant and PID controller pair of the form:

$$P(s) = \frac{N(s)}{D(s)}$$

$$C(s) = \frac{K_i + K_p s + K_d s^2}{s(1+sT)}, \quad T > 0$$

where $\deg[D(s)] = n$ and $\deg[N(s)] = m$.

Then consider the rational function

$$F(s) = s(1+sT) + \left(K_i + K_p s + K_d s^2\right) P(s)$$

Closed-loop stability is equivalent to the condition that zeros of $F(s)$ lie in the LHP. This is also equivalent to the condition

$$\sigma(F(s)) = n + 2 - (p^- - p^+)$$

PID Controller Design Continue ...

Now consider the rational function

$$\bar{F}(s) = F(s)P(-s)$$

PID Controller Design Continue ...

Now consider the rational function

$$\bar{F}(s) = F(s)P(-s)$$

Note that

$$\begin{aligned}
\sigma\left(\bar{F}(s)\right) &= \sigma(F(s)) + \sigma(P(-s)) \\
&= n + 2 - (p^- - p^+) + (z^+ - z^-) - (p^+ - p^-) \\
&= n + 2 - z^+ + z^- \\
&= \underbrace{n - m}_{\text{relative degree of } P(s)} \quad -2z^+ + 2
\end{aligned}$$

PID Controller Design Continue ...

Write

$$\begin{aligned}\bar{F}(j\omega) &= j\omega(1+j\omega T)P(-j\omega) + (K_i + j\omega K_p - \omega^2 K_d)P(j\omega)P(-j\omega) \\ &= \bar{F}_r(\omega) + j\bar{F}_i(\omega)\end{aligned}$$

where

$$\begin{aligned}\bar{F}_r(\omega) &= (K_i - K_d\omega^2)|P(j\omega)|^2 - \omega^2 T P_r(\omega) + \omega P_i(\omega) \\ \bar{F}_i(\omega) &= \omega\left(K_p|P(j\omega)|^2 + P_r(\omega) + \omega T P_i(\omega)\right)\end{aligned}$$

PID Controller Design Continue ...

Write

$$\bar{F}(j\omega) = j\omega(1+j\omega T)P(-j\omega) + (K_i + j\omega K_p - \omega^2 K_d)P(j\omega)P(-j\omega)$$
$$= \bar{F}_r(\omega) + j\bar{F}_i(\omega)$$

where

$$\bar{F}_r(\omega) = (K_i - K_d\omega^2)|P(j\omega)|^2 - \omega^2 T P_r(\omega) + \omega P_i(\omega)$$
$$\bar{F}_i(\omega) = \omega\left(K_p|P(j\omega)|^2 + P_r(\omega) + \omega T P_i(\omega)\right)$$

- K_p appears only in $\bar{F}_i(\omega)$
- K_i, K_d only in $\bar{F}_r(\omega)$

Lemma: Determining the required signature

Relative Degree and Net Phase Change:

- **A.** In the Bode magnitude plot of the LTI system $P(j\omega)$, the high frequency slope is $-(n-m)20\mathrm{dB/decade}$ where $n-m$ is the relative degree of the plant $P(s)$.

Lemma: Determining the required signature

Relative Degree and Net Phase Change:

- A. In the Bode magnitude plot of the LTI system $P(j\omega)$, the high frequency slope is $-(n-m)20\text{dB}/\text{decade}$ where $n-m$ is the relative degree of the plant $P(s)$.

- B. The net change of phase of $P(j\omega), \omega \in [0, \infty)$, denoted $\Delta_0^\infty(\phi)$ is:

$$\Delta_0^\infty(\phi) = -\left[(n-m) - 2(p^+ - z^+)\right]\frac{\pi}{2}$$

where p^+ and z^+ are numbers of RHP poles and zeros of $P(s)$, respectively.

Proof of Lemma

- The statement A is obvious from the property of Bode magnitude plot.

Proof of Lemma

- The statement A is obvious from the property of Bode magnitude plot.

- Let p^- and z^- are number of LHP poles and zeros of $P(s)$. Then the net phase change of $P(j\omega)$ for $\omega \in [0, \infty)$ is

$$\begin{aligned}\Delta_0^\infty(\phi) &= \left[(z^- - z^+) - (p^- - p^+)\right]\frac{\pi}{2} \\ &= -\left[(n-m) - 2(p^+ - z^+)\right]\frac{\pi}{2}.\end{aligned}$$

Proof of Lemma

- The statement A is obvious from the property of Bode magnitude plot.

- Let p^- and z^- are number of LHP poles and zeros of $P(s)$. Then the net phase change of $P(j\omega)$ for $\omega \in [0, \infty)$ is

$$\begin{aligned} \Delta_0^\infty(\phi) &= \left[(z^- - z^+) - (p^- - p^+)\right] \frac{\pi}{2} \\ &= -\left[(n-m) - 2(p^+ - z^+)\right] \frac{\pi}{2}. \end{aligned}$$

- From known $n-m$ from Bode plot, given p^+, and measured $\Delta_0^\infty(\phi)$, we can compute z^+.

Determining z^+ and p^+

- Assume that a known feedback controller $C(s)$ stabilizes the plant P and the closed-loop response can be *measured* and denoted by

$$G(j\omega), \quad \text{for } \omega \in [0, \infty)$$

Determining z^+ and p^+

- Assume that a known feedback controller $C(s)$ stabilizes the plant P and the closed-loop response can be *measured* and denoted by

$$G(j\omega), \quad \text{for } \omega \in [0, \infty)$$

- Then $P(j\omega)$ is the *computed* frequency response of the unstable plant

$$P(j\omega) = \frac{G(j\omega)}{C(j\omega)(1 - G(j\omega))}$$

Determining z^+ and p^+

- Assume that a known feedback controller $C(s)$ stabilizes the plant P and the closed-loop response can be *measured* and denoted by

$$G(j\omega), \quad \text{for } \omega \in [0, \infty)$$

- Then $P(j\omega)$ is the *computed* frequency response of the unstable plant

$$P(j\omega) = \frac{G(j\omega)}{C(j\omega)(1 - G(j\omega))}$$

- Knowledge of $C(s)$ and $G(j\omega)$ is sufficient to determine z^+ and p^+.

Determining z^+ and p^+ (continue...)

Lemma:

$$
\begin{aligned}
z^+ &= \frac{1}{2}\left[-r_P - r_C - 2z_c^+ - \sigma(G)\right] \\
p^+ &= \frac{1}{2}\left[\sigma(P) - \sigma(G) - r_C\right] - z_c^+
\end{aligned}
$$

where z_c^+ denotes the number of RHP zeros of $C(s)$.

Proof of Lemma

$$G(s) = \frac{P(s)C(s)}{1 + P(s)C(s)}$$

- since $G(s)$ is stable,

$$\sigma(G) = \left(z^{-} + z_c^{-}\right) - \left(z^{+} + z_c^{+}\right) - (n + n_c) = -r_P - r_C - 2z_c^{+} - 2z^{+}$$

which implies

$$z^{+} = \frac{1}{2}\left[-r_P - r_C - 2z_c^{+} - \sigma(G)\right].$$

Proof of Lemma

$$G(s) = \frac{P(s)C(s)}{1 + P(s)C(s)}$$

- since $G(s)$ is stable,

$$\sigma(G) = \left(z^- + z_c^-\right) - \left(z^+ + z_c^+\right) - (n + n_c) = -r_P - r_C - 2z_c^+ - 2z^+$$

  which implies

$$z^+ = \frac{1}{2}\left[-r_P - r_C - 2z_c^+ - \sigma(G)\right].$$

- From $\sigma(P)$ applied to $P(s)$, we have $p^+ = z^+ + \frac{1}{2}\sigma(P) + \frac{1}{2}r_P$.
  Then we now have

$$p^+ = \frac{1}{2}\left[\sigma(P) - \sigma(G) - r_C\right] - z_c^+.$$

Determining the zeros of $\bar{F}_i(\omega) = 0$

Recall

$$\bar{F}_i(\omega) = \omega(K_p|P(j\omega)|^2 + P_r(\omega) + \omega T P_i(\omega)) = 0$$

Determining the zeros of $\bar{F}_i(\omega) = 0$

Recall

$$\bar{F}_i(\omega) = \omega(K_p|P(j\omega)|^2 + P_r(\omega) + \omega T P_i(\omega)) = 0$$

Then for $\omega \neq 0$,

$$K_p = -\frac{P_r(\omega) + \omega T P_i(\omega)}{|P(j\omega)|^2}$$

or

$$K_p = -\frac{\cos\phi(\omega) + \omega T \sin\phi(\omega)}{|P(j\omega)|}$$

PID Controller Design Continue ...

The set of PID stabilizing controllers can be found as follows:

- Fix $K = K_p^*$ and set $\bar{F}_i(\omega, K_p^*) = 0$.

PID Controller Design Continue ...

The set of PID stabilizing controllers can be found as follows:

- Fix $K = K_p^*$ and set $\bar{F}_i(\omega, K_p^*) = 0$.
- Let $i_t \in \{+1, 0, -1\}$ and $j \in \{+1, -1\}$.

PID Controller Design Continue ...

The set of PID stabilizing controllers can be found as follows:

- Fix $K = K_p^*$ and set $\bar{F}_i(\omega, K_p^*) = 0$.
- Let $i_t \in \{+1, 0, -1\}$ and $j \in \{+1, -1\}$.
- Determine strings of integers $\{i_0, i_1, \cdots\}$ satisfying:

PID Controller Design Continue ...

The set of PID stabilizing controllers can be found as follows:

- Fix $K = K_p^*$ and set $\bar{F}_i(\omega, K_p^*) = 0$.
- Let $i_t \in \{+1, 0, -1\}$ and $j \in \{+1, -1\}$.
- Determine strings of integers $\{i_0, i_1, \cdots\}$ satisfying:
  - For $n - m$ even:

$$\left\{i_0 - 2i_1 + 2i_2 + \cdots + (-1)^{i-1} 2i_{l-1} + (-1)^l i_l\right\} \cdot (-1)^{l-1} j$$
$$= n - m + 2z^+ + 2$$

PID Controller Design Continue ...

The set of PID stabilizing controllers can be found as follows:

- Fix $K = K_p^*$ and set $\bar{F}_i(\omega, K_p^*) = 0$.
- Let $i_t \in \{+1, 0, -1\}$ and $j \in \{+1, -1\}$.
- Determine strings of integers $\{i_0, i_1, \cdots\}$ satisfying:
  - For $n - m$ even:

$$\left\{i_0 - 2i_1 + 2i_2 + \cdots + (-1)^{i-1} 2i_{l-1} + (-1)^l i_l\right\} \cdot (-1)^{l-1} j$$
$$= n - m + 2z^+ + 2$$

  - For $n - m$ odd:

$$\{i_0 - 2i_1 + 2i_2 + \cdots + (-1)^{i-1} 2i_{l-1}\} \cdot (-1)^{l-1} j$$
$$= n - m + 2z^+ + 2.$$

PID Controller Design Continue ...

- For each string (sign sequence of appropriate number of terms) satisfying the signature formula, the conditions for stability are:

$$\operatorname{sgn}\left[R(\omega_t, K_i, K_d)\right] i_t > 0, \quad \text{for} \quad t = 0, 1, 2, \cdots,$$

PID Controller Design Continue ...

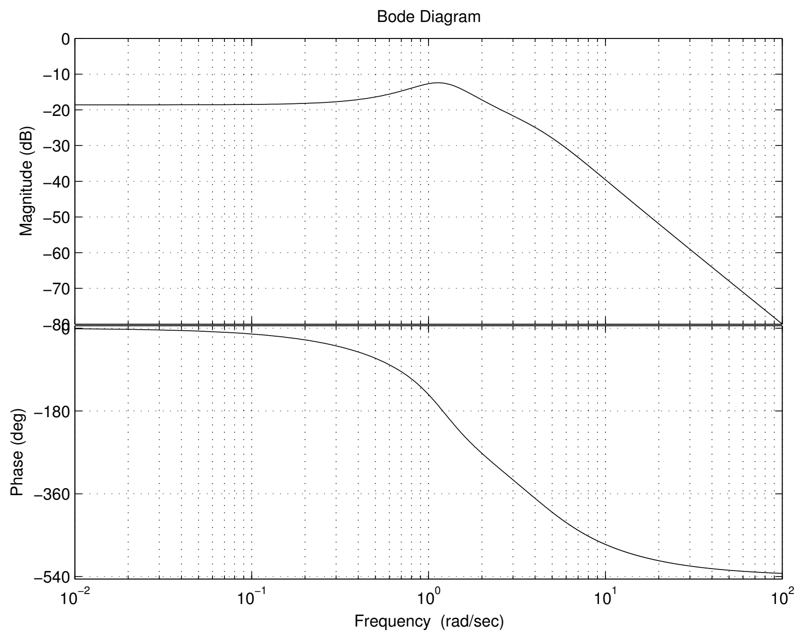
- For each string (sign sequence of appropriate number of terms) satisfying the signature formula, the conditions for stability are:

$$\operatorname{sgn}\left[R(\omega_t, K_i, K_d)\right] i_t > 0, \quad \text{for} \;\; t = 0, 1, 2, \cdots,$$

- Each valid string produces a set of linear inequalities in (K_i, K_d) space.

Example

Consider the *stable* plant



$$-6\frac{\pi}{2} = -\Bigg(\underbrace{(n-m)}_{=2} - 2(\underbrace{p^+}_{=0} - z^+)\Bigg)\frac{\pi}{2} \Rightarrow z^+ = 2$$

Example (continue...)

- The required signature for stability can now be determined and is

$$\sigma(\bar{F}) = (n - m) + 2z^{+} + 2 = (2) + 2(2) + 2 = 8.$$

Example (continue...)

- The required signature for stability can now be determined and is

$$\sigma(\bar{F}) = (n-m) + 2z^{+} + 2 = (2) + 2(2) + 2 = 8.$$

- Since $n-m$ is even, we have

$$i_0 - 2i_1 + 2i_2 - 2i_3 + 2i_4 - \cdots + (-1)^l i_l = 8,$$

and it is clear that at least four terms are required to satisfy the above.

Example (continue...)

- The required signature for stability can now be determined and is

$$\sigma(\bar{F}) = (n-m) + 2z^{+} + 2 = (2) + 2(2) + 2 = 8.$$

- Since $n-m$ is even, we have

$$i_0 - 2i_1 + 2i_2 - 2i_3 + 2i_4 - \cdots + (-1)^l i_l = 8,$$

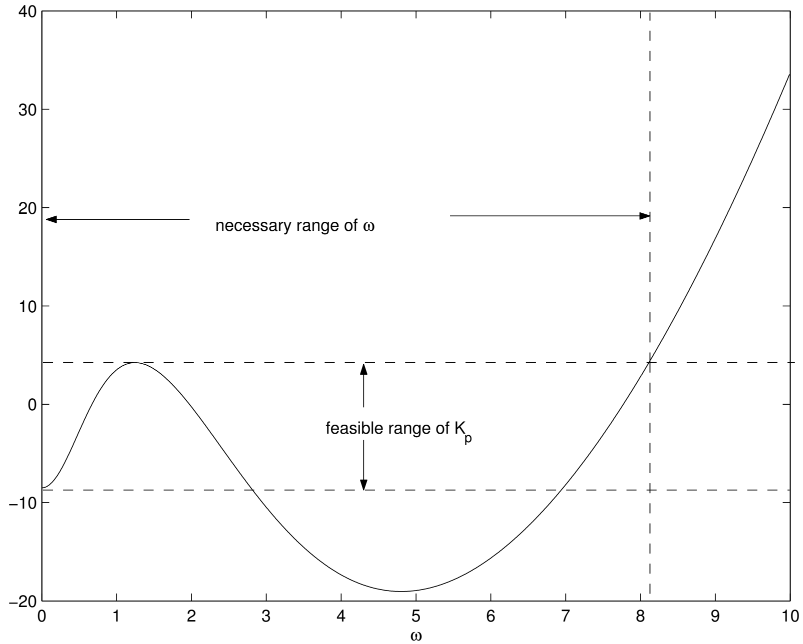
  and it is clear that at least four terms are required to satisfy the above.

- In other words $l \geq 4$.

Example (continue...)

From the figure it is easy to see that K_p^* has at most three positive frequencies as solutions and therefore we have

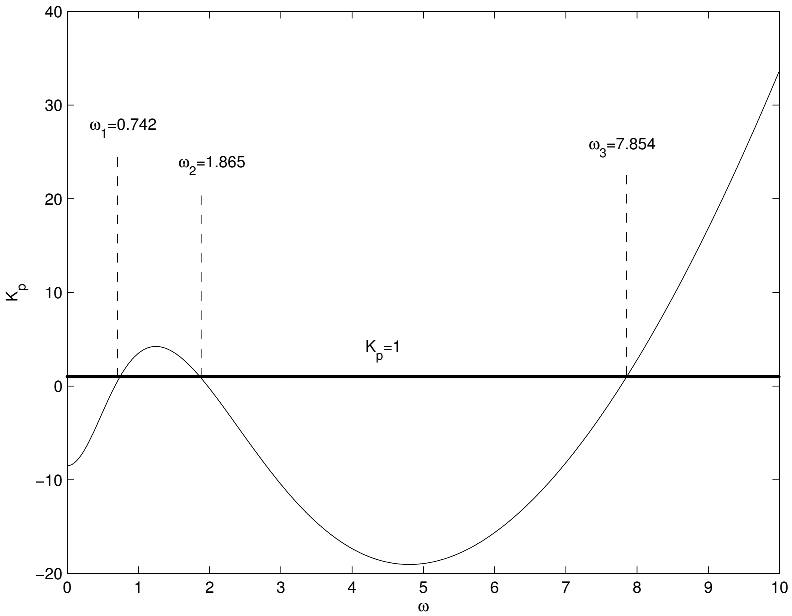
$$i_0 - 2i_1 + 2i_2 - 2i_3 + i_4 = 8.$$



Example (continue...)

Fix $K_p = 1$ and compute the set of ω's that satisfies

$$-\frac{\cos\phi(\omega) + \omega T \sin\phi(\omega)}{|P(j\omega)|} = 1.$$



Example (continue...)

- This leads to the requirement

$$i_0 - 2i_1 + 2i_2 - 2i_3 = 7$$

Example (continue...)

- This leads to the requirement

$$i_0 - 2i_1 + 2i_2 - 2i_3 = 7$$

- giving the feasible string

$$\mathcal{F} = \{i_0, i_1, i_2, i_3\} = \{1, \ -1, \ 1, \ -1\}.$$

Example (continue...)

- This leads to the requirement

$$i_0 - 2i_1 + 2i_2 - 2i_3 = 7$$

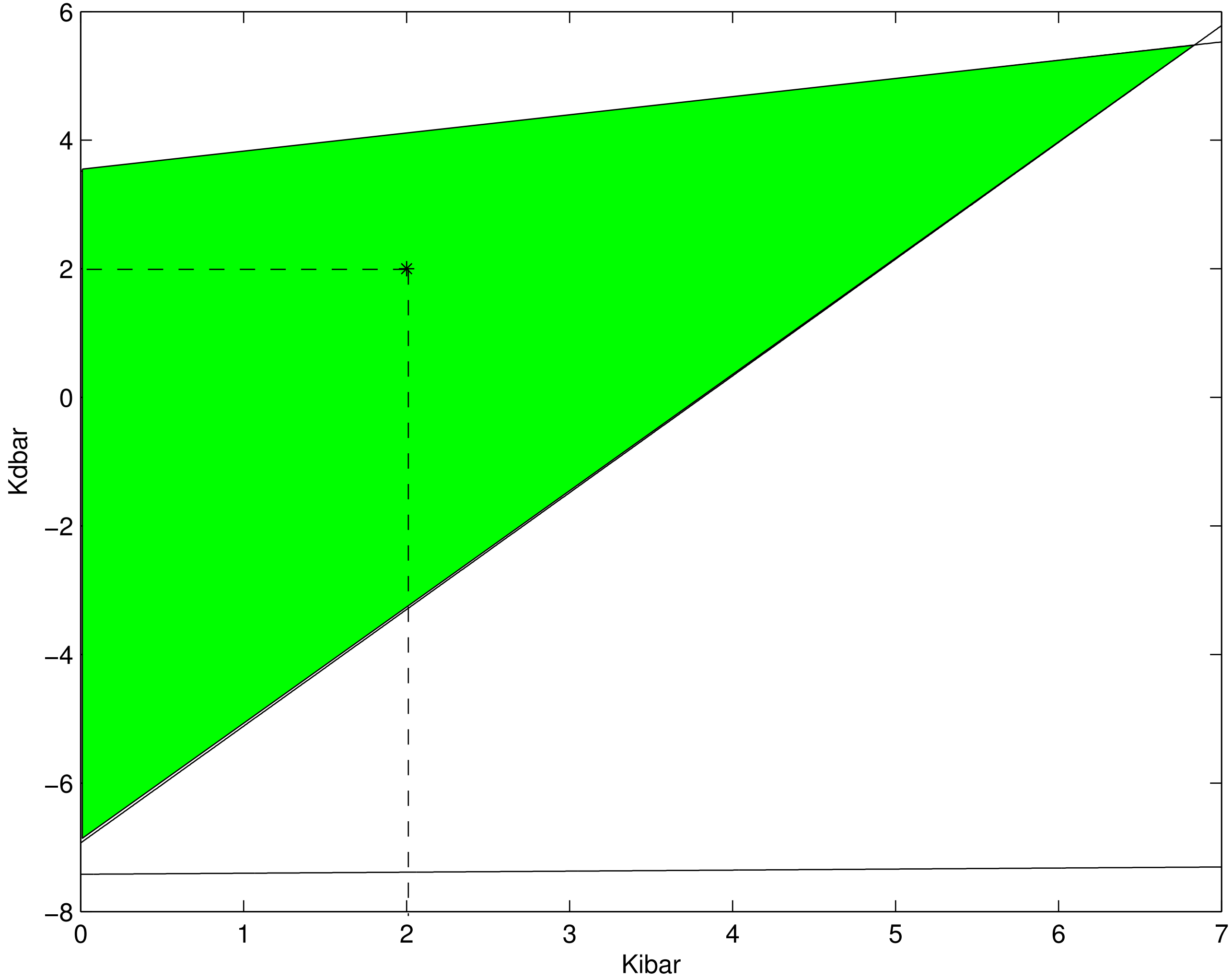
- giving the feasible string

$$\mathcal{F} = \{i_0, i_1, i_2, i_3\} = \{1,\ -1,\ 1,\ -1\}.$$

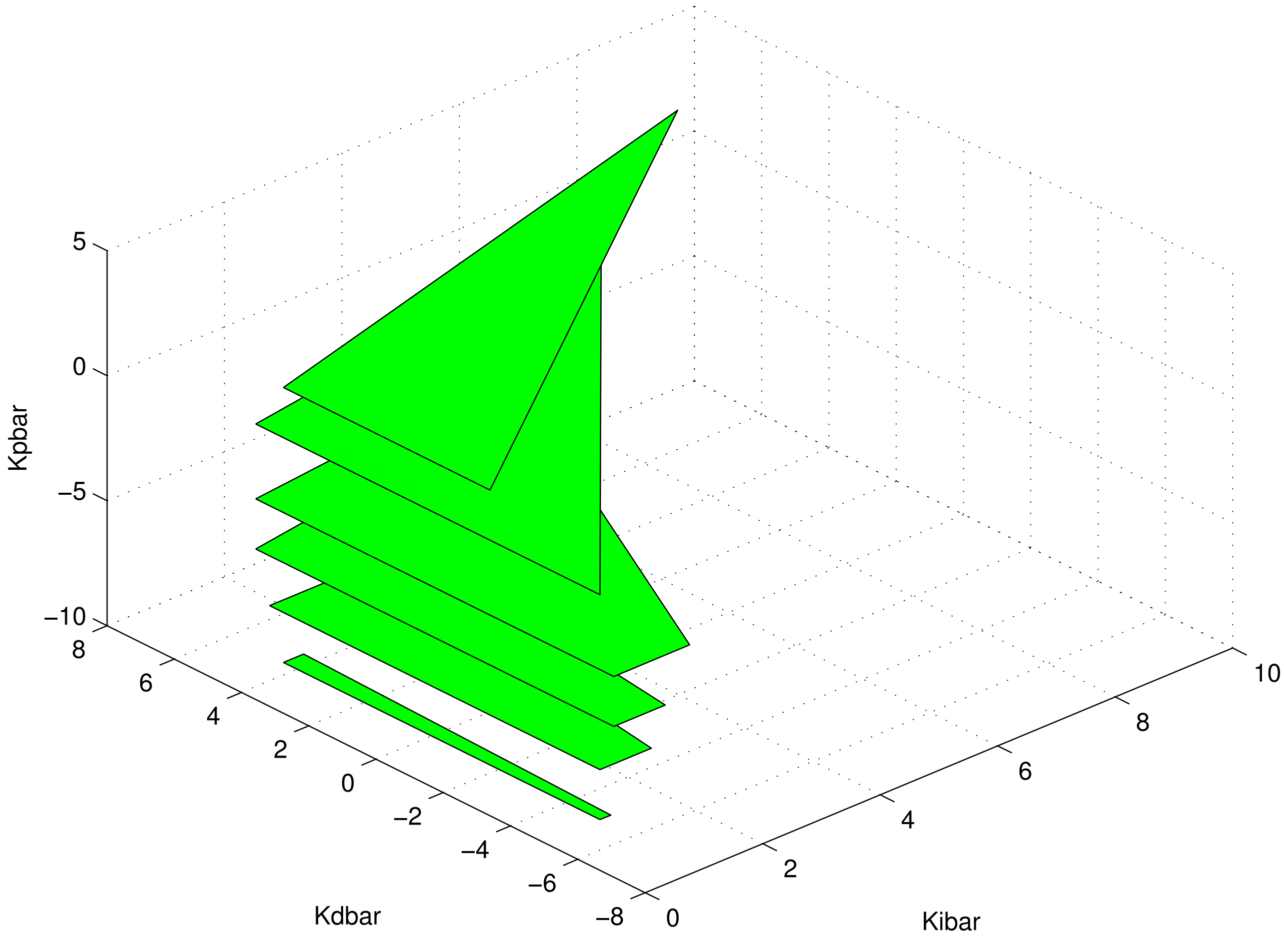
- Thus, we have the following set of linear inequalities for stability:

$$\begin{aligned} 0.0138K_i &> 0 \\ -0.1390 + 0.0364K_i - 0.0201K_d &< 0 \\ 0.2791 + 0.0229K_i - 0.0797K_d &> 0 \\ -0.1349 + 0.0003K_i - 0.0182K_d &< 0 \end{aligned}$$

Example: Stabilizing PID Set for $K_p = 1$



Example: Entire Stabilizing PID Set



Performance Specifications

Many performance attainment problems can be cast as stabilization of families of real and complex plants. For example,

- The problem of achieving a gain margin is equivalent to stabilizing the family of *real* plants

$$\mathcal{P}^c(s) = \{KP(s) : K \in [K_{\min}, K_{\max}]\} .$$

Performance Specifications

Many performance attainment problems can be cast as stabilization of families of real and complex plants. For example,

- The problem of achieving a gain margin is equivalent to stabilizing the family of *real* plants

$$\mathcal{P}^c(s) = \{KP(s) : K \in [K_{\min}, K_{\max}]\} .$$

- The problem of achieving prescribed phase margin θ_m is equivalent to stabilizing the family of *complex* plants

$$\mathcal{P}^c(s) = \left\{e^{-j\theta} P(s) : \theta \in [0, \theta_m]\right\} .$$

- The problem of achieving an H_∞ norm specification on the sensitivity function $S(s)$, that is, $\|W(s)S(s)\|_\infty < \gamma$ is equivalent to stabilizing the family of *complex* plants

$$\mathcal{P}^c(s) = \left\{ \left[\frac{1}{1 + \frac{1}{\gamma} e^{j\theta} W(s)} \right] P(s) : \theta \in [0, 2\pi] \right\}.$$

- The problem of achieving an H_∞ norm specification on the sensitivity function $S(s)$, that is, $\|W(s)S(s)\|_\infty < \gamma$ is equivalent to stabilizing the family of *complex* plants

$$\mathcal{P}^c(s) = \left\{ \left[\frac{1}{1 + \frac{1}{\gamma} e^{j\theta} W(s)} \right] P(s) : \theta \in [0, 2\pi] \right\}.$$

- The problem of achieving an H_∞ norm specification on the complementary sensitivity function $T(s)$, that is, $W(s)T(s)\|_\infty < \gamma$ is equivalent to stabilizing the family of *complex* plants

$$\mathcal{P}^c(s) = \left\{ P(s) \left[1 + \frac{1}{\gamma} e^{j\theta} W(s) \right] : \theta \in [0, 2\pi] \right\}.$$

Assumption

The only information available to the designer is:

- Knowledge of the frequency response magnitude and phase, equivalently, $P^c(j\omega)$, $\omega \in (-\infty, +\infty)$.
- Knowledge of the number of RHP poles, p^+.

Determining Performance Set

- The complete set of stabilizing PID gains for a given complex LTI plant can be found from the frequency response data $P^c(j\omega)$ and the knowledge of the number of RHP poles

Determining Performance Set

- The complete set of stabilizing PID gains for a given complex LTI plant can be found from the frequency response data $P^c(j\omega)$ and the knowledge of the number of RHP poles

- The set of stabilizing PID gains can be computed by the following procedure:

Determining Performance Set

- The complete set of stabilizing PID gains for a given complex LTI plant can be found from the frequency response data $P^c(j\omega)$ and the knowledge of the number of RHP poles

- The set of stabilizing PID gains can be computed by the following procedure:
  - Determine the relative degree $n_c - m_c$ from the high frequency slope of the Bode magnitude plot where n_c and m_c are degrees of numerator and denominator of P_c.

Determining Performance Set

- The complete set of stabilizing PID gains for a given complex LTI plant can be found from the frequency response data $P^c(j\omega)$ and the knowledge of the number of RHP poles

- The set of stabilizing PID gains can be computed by the following procedure:

  - Determine the relative degree $n_c - m_c$ from the high frequency slope of the Bode magnitude plot where n_c and m_c are degrees of numerator and denominator of P_c.

  - Fix $K_p = K_p^*$ and solve

$$K_p^* = -\frac{\cos\phi(\omega) + \omega T \sin\phi(\omega)}{|P^c(j\omega)|}$$

    and let $\omega_1 < \omega_2 < \cdots < \omega_{l-1}$ denote the distinct frequencies which are solutions of the above.

- Set $\omega_0 = -\infty, \omega_l = +\infty$ and determine all strings of integers $i_t \in \{+1, 0, -1\}$ and $j \in \{-1, +1\}$ such that

$$\sum_{r=1}^{l-1} (-1)^{l-1-r} i_r \cdot j = n_c - m_c + 2z_c^+ + 2$$

where n_c and m_c denote the numerator and denominator degrees of $P^c(s)$ and z_c^+ the number of RHP zeros.

- Set $\omega_0 = -\infty$, $\omega_l = +\infty$ and determine all strings of integers $i_t \in \{+1, 0, -1\}$ and $j \in \{-1, +1\}$ such that

$$\sum_{r=1}^{l-1} (-1)^{l-1-r} i_r \cdot j = n_c - m_c + 2z_c^+ + 2$$

  where n_c and m_c denote the numerator and denominator degrees of $P^c(s)$ and z_c^+ the number of RHP zeros.

- For the fixed $K_p = K_p^*$ chosen in Step 1, solve for the stabilizing (K_i, K_d) from:

$$\left[K_i - K_d \omega_t^2 + \frac{\omega_t \sin\phi(\omega_t) - \omega_t^2 T \cos\phi(\omega_t)}{|P^c(j\omega)|} \right] i_t > 0$$

  for $t = 0, 1, \cdots$.

- Set $\omega_0 = -\infty$, $\omega_l = +\infty$ and determine all strings of integers $i_t \in \{+1, 0, -1\}$ and $j \in \{-1, +1\}$ such that

$$\sum_{r=1}^{l-1} (-1)^{l-1-r} i_r \cdot j = n_c - m_c + 2z_c^+ + 2$$

where n_c and m_c denote the numerator and denominator degrees of $P^c(s)$ and z_c^+ the number of RHP zeros.

- For the fixed $K_p = K_p^*$ chosen in Step 1, solve for the stabilizing (K_i, K_d) from:

$$\left[K_i - K_d\omega_t^2 + \frac{\omega_t \sin\phi(\omega_t) - \omega_t^2 T \cos\phi(\omega_t)}{|P^c(j\omega)|}\right] i_t > 0$$

for $t = 0, 1, \cdots$.

- Repeat the previous three steps by updating K_p over prescribed ranges.

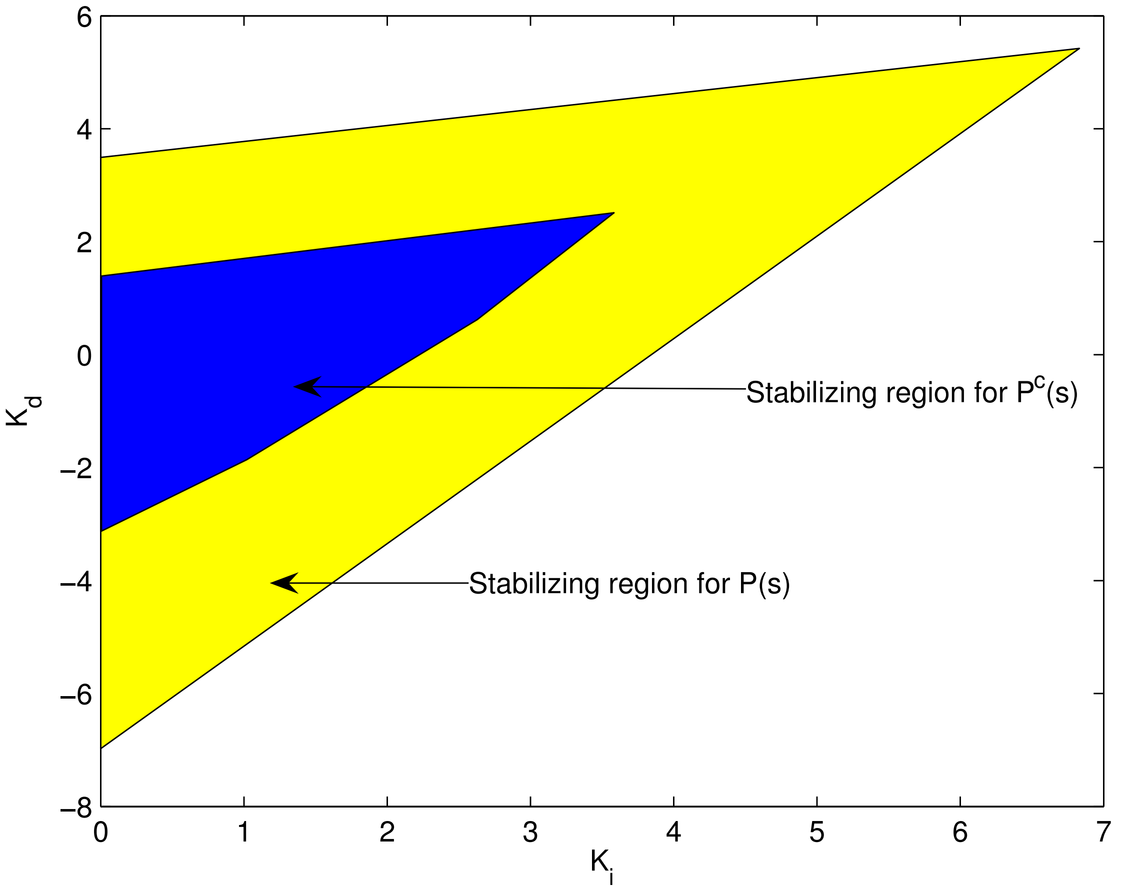
Performance Example

Taking the same frequency domain data set $\mathbf{P}(j\omega)$ used in the previous example, we consider the problem of achieving an H_∞ norm specification on the complementary sensitivity function $T(s)$, that is,

$$\|W(s)T(s)\|_\infty < 1 \quad \text{where } W(s) = \frac{s+0.1}{s+1}.$$

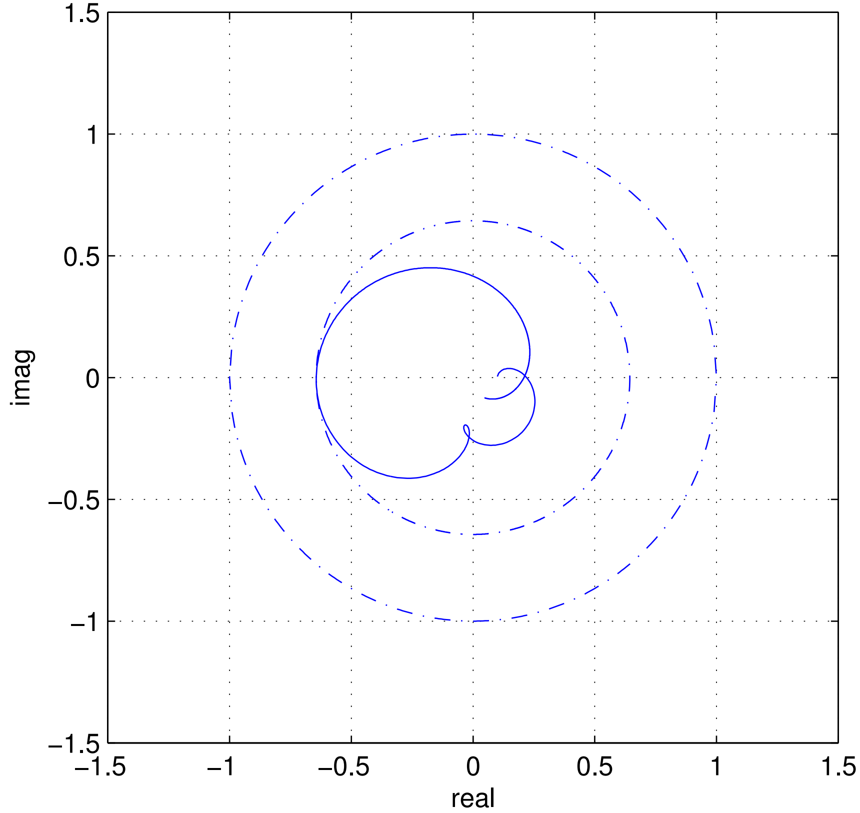
By solving the complex stabilization problem, we have the stabilizing PID controller parameter region that satisfies the given H_∞ norm specification.

The complete set of Stabilizing PID gains for H_∞ specification when $K_p = 1$



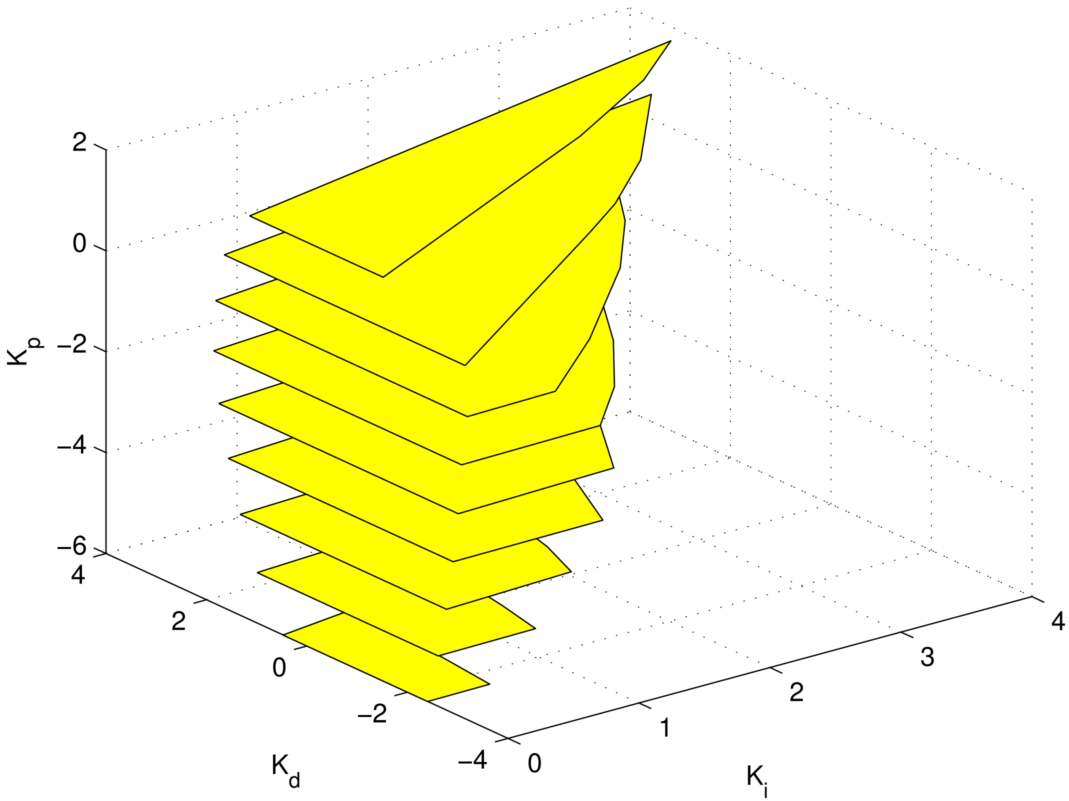
Nyquist plot of $W(s)T(s)$

By selecting a point, we verify that the point selected satisfied the given H_∞ specification.



Entire set of stabilizing PID gains satisfying the H_∞ specification

By sweeping K_p, we have the entire stabilizing PID gains that satisfy the given H_∞ specification as shown in Figure.



Example with Gain/Phase Margin Specification

Consider the following nonminimum phase plant:

$$P(s) = \left[\frac{s^3 - 4s^2 + s + 2}{s^5 + 8s^4 + 32s^3 + 46s^2 + 46s + 17} \right] e^{-s}$$

Example with Gain/Phase Margin Specification

Consider the following nonminimum phase plant:

$$P(s) = \left[\frac{s^3 - 4s^2 + s + 2}{s^5 + 8s^4 + 32s^3 + 46s^2 + 46s + 17} \right] e^{-s}$$

We wish to find the entire set of controllers that simultaneously satisfies the following specifications:

- PID controllers must stabilize the given plant with a delay
- The closed-loop system must guarantee the following gain and phase margins:

Example with Gain/Phase Margin Specification

Consider the following nonminimum phase plant:

$$P(s) = \left[\frac{s^3 - 4s^2 + s + 2}{s^5 + 8s^4 + 32s^3 + 46s^2 + 46s + 17}\right] e^{-s}$$

We wish to find the entire set of controllers that simultaneously satisfies the following specifications:

- PID controllers must stabilize the given plant with a delay
- The closed-loop system must guarantee the following gain and phase margins:
  - Gain margin : $K^+ \geq 2$ (about $6\ [dB]$)

Example with Gain/Phase Margin Specification

Consider the following nonminimum phase plant:

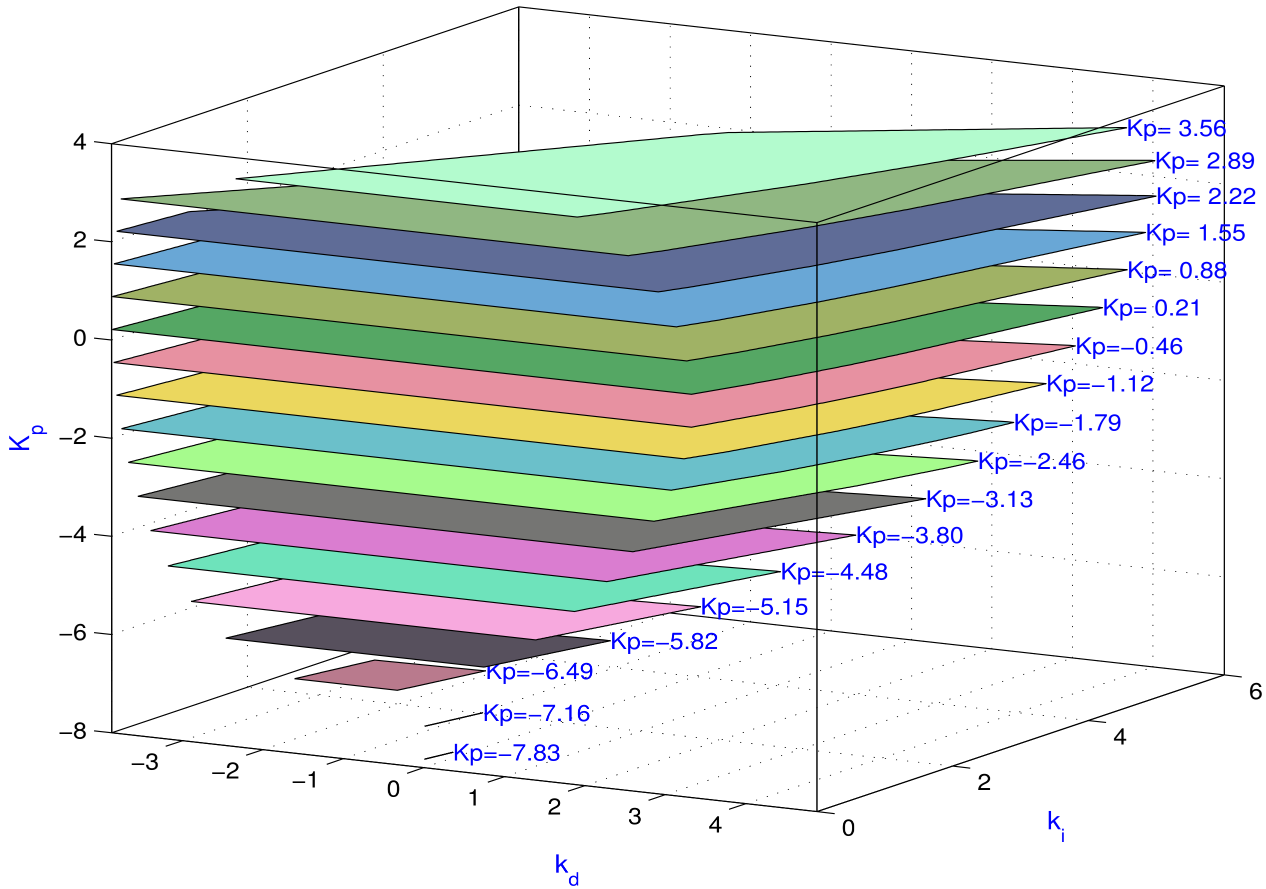
$$P(s) = \left[\frac{s^3 - 4s^2 + s + 2}{s^5 + 8s^4 + 32s^3 + 46s^2 + 46s + 17}\right] e^{-s}$$

We wish to find the entire set of controllers that simultaneously satisfies the following specifications:

- PID controllers must stabilize the given plant with a delay
- The closed-loop system must guarantee the following gain and phase margins:
  - Gain margin : $K^+ \geq 2$ (about $6\ [dB]$)
  - Phase margin : $[\theta^-, \theta^+] = [-10^o, 60^o]$

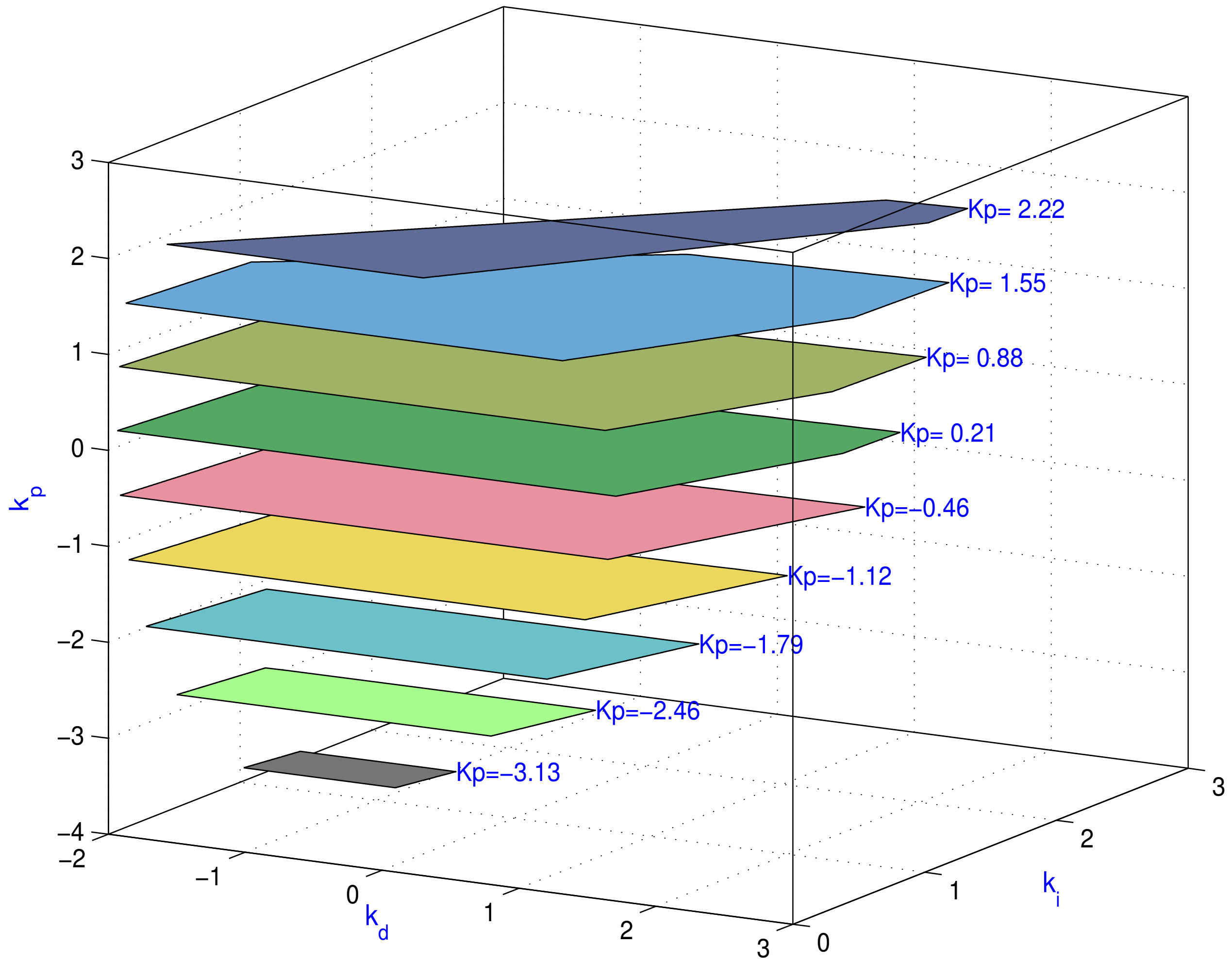
(A) All Stabilizing PID Controllers

The feasible ranges of k_p given are: $[-19.1, -8.5]$, $[-8.5, 4.23]$, $[4, 23, \infty]$.
We can easily conclude that the only region that contains the solutions is $[-8.5, 4.23]$.
The complete set of stabilizing PID controllers is:



(B) All PID Satisfying the GM and PM Requirement

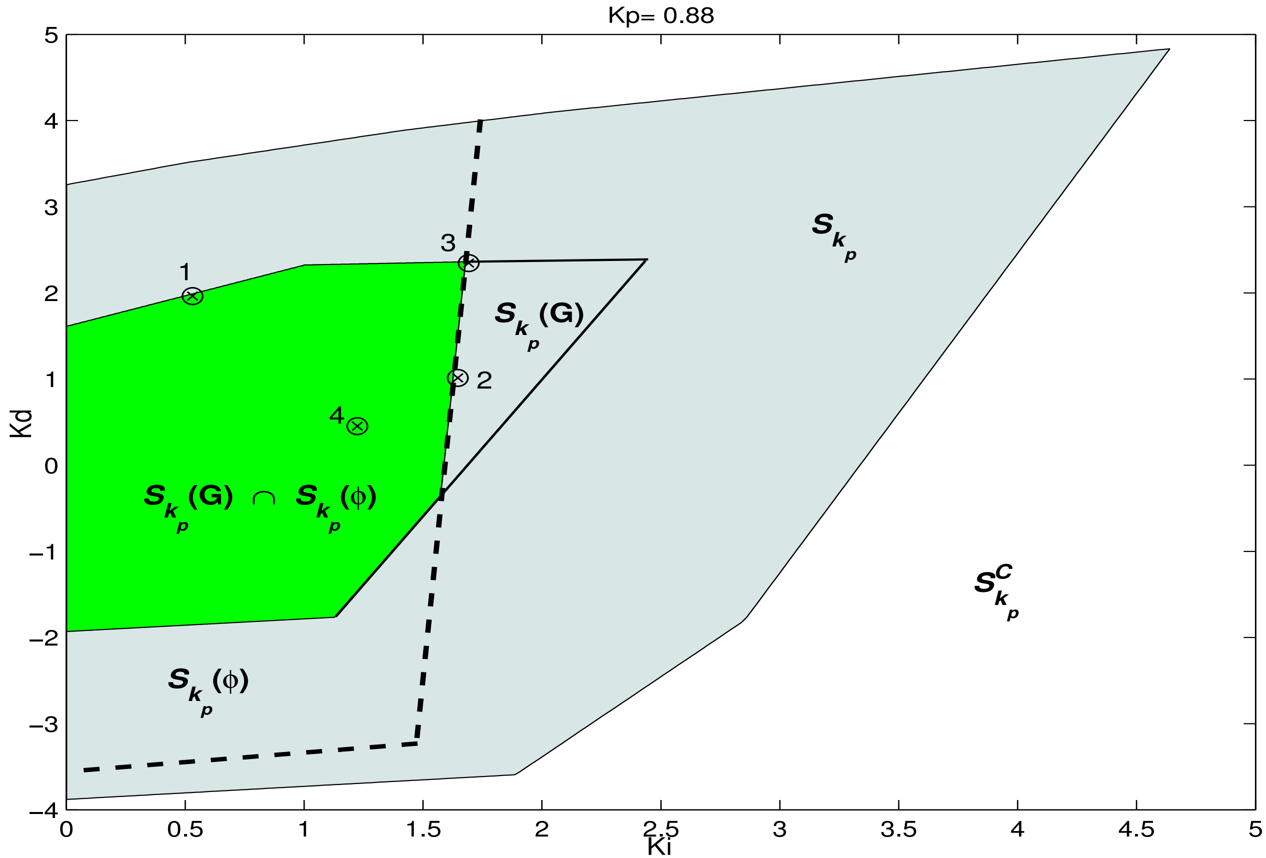
We obtained the final result.



2-D Regions with $k_p = 0.88$

$\mathcal{S}_{k_p}$	: stable set with a fixed k_p	$\mathcal{S}_{k_p}^C$	: unstable set
$\mathcal{S}_{k_p}(G)$	: set satisfying the GM requirement	$\mathcal{S}_{k_p}(\phi)$	: set satisfying the PM requiremen

$\mathcal{S}_{k_p}(G) \cap \mathcal{S}_{k_p}(\phi)$ set satisfying both gain and phase margin requirements



First Order Controller Design

First Order Controller Design

Consider the First order controller of the form

$$C(s) = \frac{x_1 s + x_2}{s + x_3}$$

for an LTI plant.

First Order Controller Design

Consider the First order controller of the form

$$C(s) = \frac{x_1 s + x_2}{s + x_3}$$

for an LTI plant.

Assumption

- The plant has no $j\omega$ poles or zeros.
- Available frequency domain data $P(j\omega)$ for $\omega \in [0, \infty)$.
- Knowledge of p^+.

Root Invariant Regions

- Let

$$
\begin{aligned}
P(j\omega) &= P_r(\omega) + jP_i(\omega) \\
F(s) &= (s + x_3) + (sx_1 + x_2)P(s)
\end{aligned}
$$

Root Invariant Regions

- Let

$$\begin{aligned} P(j\omega) &= P_r(\omega) + jP_i(\omega) \\ F(s) &= (s + x_3) + (sx_1 + x_2)P(s) \end{aligned}$$

- For closed-loop stability, it is necessary and sufficient that

$$\sigma(F(s)) = n + 1 - (p^- - p^+)$$

Root Invariant Regions

- Let

$$
\begin{aligned}
P(j\omega) &= P_r(\omega) + jP_i(\omega) \\
F(s) &= (s + x_3) + (sx_1 + x_2)P(s)
\end{aligned}
$$

- For closed-loop stability, it is necessary and sufficient that

$$\sigma(F(s)) = n + 1 - (p^- - p^+)$$

- Let

$$\bar{F}(s) = F(s)P(-s)$$

and the stability condition is

$$\bar{F}(s) = (s + x_3)P(-s) + (sx_1 + x_2)P(s)P(-s)$$

Root Invariant Region (continue...)

- In other words,

$$
\begin{aligned}
\bar{F}(\jmath\omega, x_1, x_2, x_3) \quad &= \quad \underbrace{x_2|P(\jmath\omega)|^2 + \omega P_i(\omega) + x_3 P_r(\omega)}_{\bar{F}_r(\omega, x_1, x_2, x_3)} \\
&\quad + \jmath\omega \underbrace{\left(x_1|P(\jmath\omega)|^2 - x_3 P_i(\omega) + P_r(\omega)\right)}_{\bar{F}_i(\omega, x_1, x_2, x_3)}.
\end{aligned}
$$

Root Invariant Region (continue...)

- In other words,

$$\bar{F}(\jmath\omega, x_1, x_2, x_3) = \underbrace{x_2|P(\jmath\omega)|^2 + \omega P_i(\omega) + x_3 P_r(\omega)}_{\bar{F}_r(\omega, x_1, x_2, x_3)} + \jmath\omega \underbrace{\left(x_1|P(\jmath\omega)|^2 - x_3 P_i(\omega) + P_r(\omega)\right)}_{\bar{F}_i(\omega, x_1, x_2, x_3)}.$$

- The curves $\bar{F}_r(\omega, \cdot) = 0$ and $\bar{F}_i(\omega, \cdot) = 0$, $0 \leq \omega < \infty$ along with the $\bar{F}(0, \cdot) = 0$ and $\bar{F}(\infty, \cdot) = 0$ partition the (x_1, x_2, x_3) parameter space into signature invariant regions.

Root Invariant Region (continue...)

- In other words,

$$\begin{aligned}\bar{F}(\jmath\omega, x_1, x_2, x_3) &= \underbrace{x_2|P(\jmath\omega)|^2 + \omega P_i(\omega) + x_3 P_r(\omega)}_{\bar{F}_r(\omega, x_1, x_2, x_3)} \\ &\quad + \jmath\omega \underbrace{\left(x_1|P(\jmath\omega)|^2 - x_3 P_i(\omega) + P_r(\omega)\right)}_{\bar{F}_i(\omega, x_1, x_2, x_3)}.\end{aligned}$$

- The curves $\bar{F}_r(\omega, \cdot) = 0$ and $\bar{F}_i(\omega, \cdot) = 0$, $0 \leq \omega < \infty$ along with the $\bar{F}(0, \cdot) = 0$ and $\bar{F}(\infty, \cdot) = 0$ partition the (x_1, x_2, x_3) parameter space into signature invariant regions.

- By plotting these curves and selecting a test point from each of these regions we can determine the stability regions corresponding to those with signature equal to $n - m + 2z^+ + 1$.

Procedure for First Order Controller Design

- 1. Determine the relative degree $n - m$ from the high frequency slope of the Bode magnitude plot.

Procedure for First Order Controller Design

- 1. Determine the relative degree $n-m$ from the high frequency slope of the Bode magnitude plot.

- 2. Let $\Delta_0^\infty[\phi(\omega)]$ denote the net change of phase $P(\jmath\omega)$ for $\omega \in [0,\infty)$. Determine z^+ from knowledge of p^+ and

$$\Delta_0^\infty[\phi(\omega)] = -\left[(n-m) - 2(p^+ - z^+)\right]\frac{\pi}{2}.$$

Procedure for First Order Controller Design

- 1. Determine the relative degree $n-m$ from the high frequency slope of the Bode magnitude plot.

- 2. Let $\Delta_0^\infty[\phi(\omega)]$ denote the net change of phase $P(\jmath\omega)$ for $\omega \in [0,\infty)$. Determine z^+ from knowledge of p^+ and

$$\Delta_0^\infty[\phi(\omega)] = -\left[(n-m) - 2(p^+ - z^+)\right]\frac{\pi}{2}.$$

- 3. Plot the curves below in the (x_1, x_2) plane for a fixed x_3.

$$x_3 + x_2 P(0) = 0$$

$$\begin{cases} x_1(\omega) = \frac{1}{|P(\jmath\omega)|}\left(\frac{\sin\phi(\omega)}{\omega}x_3 - \cos\phi(\omega)\right), & \text{for } 0 < \omega < \infty \\ x_2(\omega) = -\frac{1}{|P(\jmath\omega)|}\left(\cos\phi(\omega)x_3 + \omega\sin\phi(\omega)\right), & \text{for } 0 < \omega < \infty \end{cases}$$

$$1 + P(\infty)x_2 = 0.$$

Procedure for First Order Controller Design (continue...)

- 4. The curves $x_1(\omega)$ and $x_2(\omega)$ partition the (x_1, x_2) plane into disjoint signature invariant regions. The stabilizing regions correspond to those for which $\bar{F}(s)$ has a signature of $n - m + 2z^+ + 1$.

Example

- For illustration, we have collected the frequency domain (Nyquist-Bode) data of a stable plant:

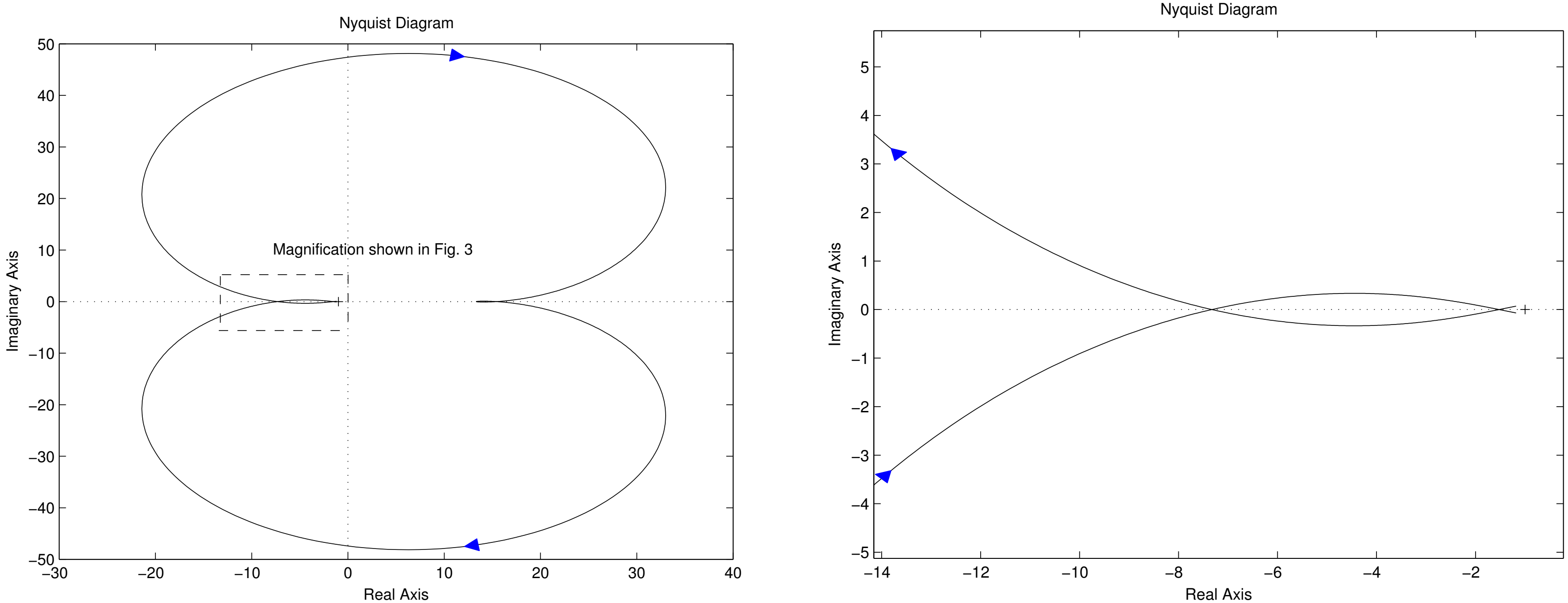
$$\mathbf{P}(\jmath\omega) = \{P(\jmath\omega) : \omega \in (0, 10) \text{ sampled every } 0.01\}.$$

Example

- For illustration, we have collected the frequency domain (Nyquist-Bode) data of a stable plant:

$$\mathbf{P}(\jmath\omega) = \{P(\jmath\omega) : \omega \in (0, 10) \text{ sampled every } 0.01\}.$$

- The Nyquist plot of the plant obtained is:



Example (continue...)

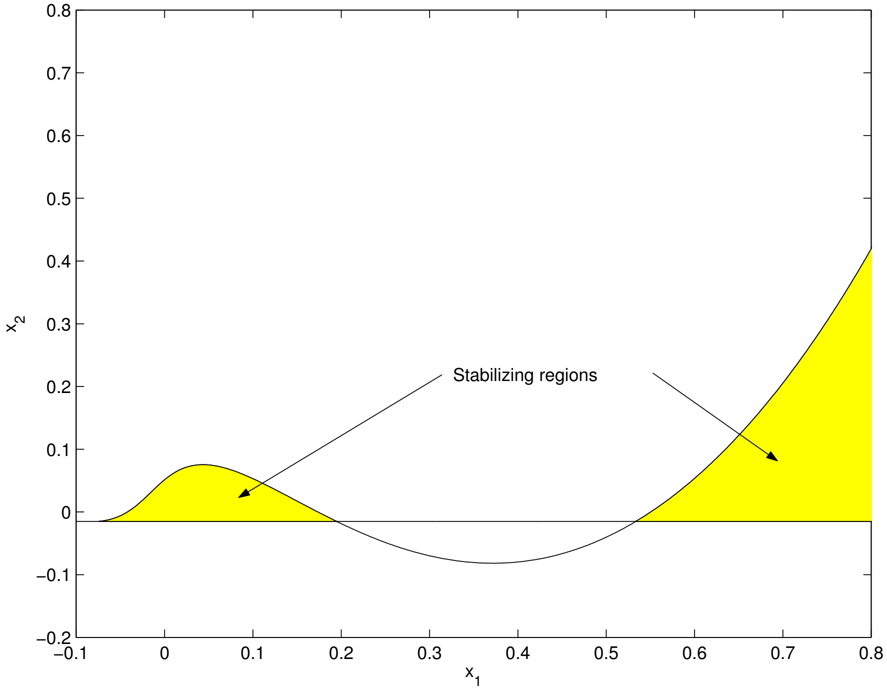
- From the data $\mathbf{P}(\jmath\omega)$, we have $P(0) = 13.333$ and $P(\infty) = 0$. Then it is easy to see that the straight line is not applicable.

Example (continue...)

- From the data $\mathbf{P}(\jmath\omega)$, we have $P(0) = 13.333$ and $P(\infty) = 0$. Then it is easy to see that the straight line is not applicable.
- After fixing $x_3 = 0.2$, the data points representing the straight line and the curve are depicted.

Example (continue...)

- From the data $\mathbf{P}(\jmath\omega)$, we have $P(0) = 13.333$ and $P(\infty) = 0$. Then it is easy to see that the straight line is not applicable.
- After fixing $x_3 = 0.2$, the data points representing the straight line and the curve are depicted.
- By testing a point for each root invariant region, we have the following.



Example (continue...)

- We now consider the problem of determining the entire set of first order stabilizing controllers satisfying the required closed-loop performance described by the requirement on the H_∞ norm of the weighted complementary sensitivity function:

$$\|W(\jmath\omega)T(\jmath\omega)\|_\infty < \gamma, \quad \text{for all } \omega$$

Example (continue...)

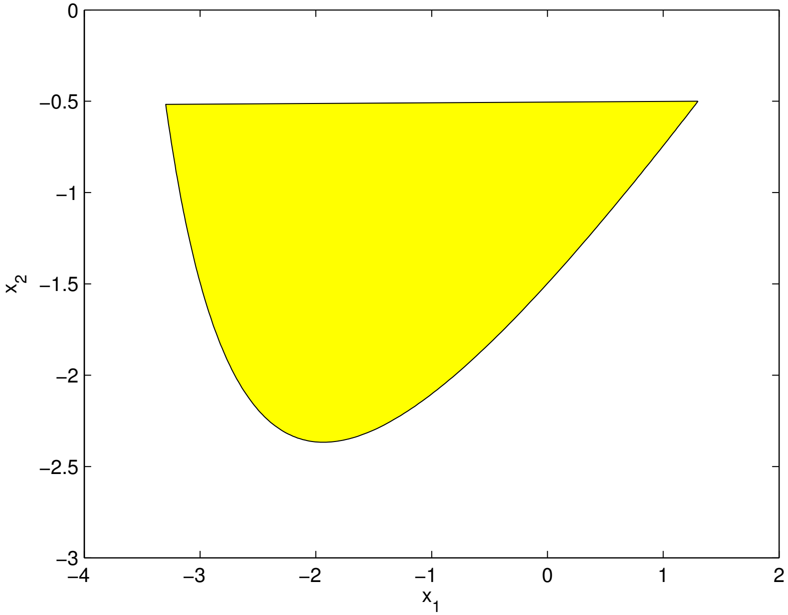
- We now consider the problem of determining the entire set of first order stabilizing controllers satisfying the required closed-loop performance described by the requirement on the H_∞ norm of the weighted complementary sensitivity function:

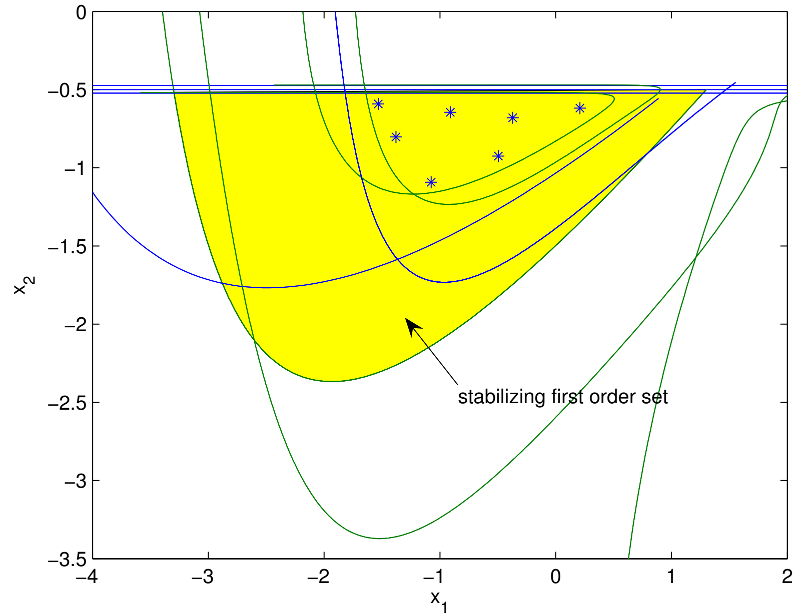
$$\|W(\jmath\omega)T(\jmath\omega)\|_\infty < \gamma, \quad \text{for all } \omega$$

- This is equivalent to the problem of simultaneously stabilizing the specified complex family as well as the original plant $P(s)$.

Example (continue...)

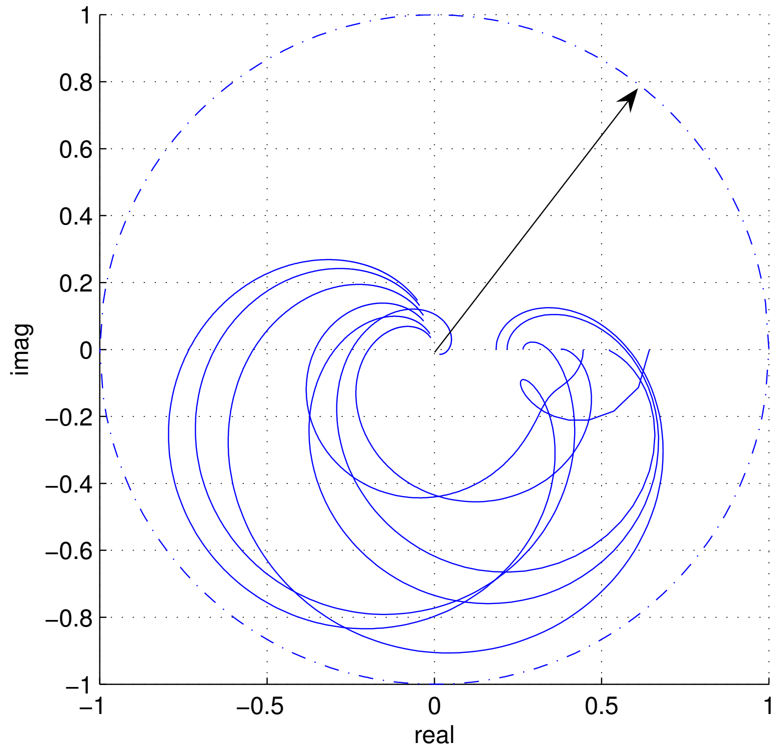
- In this problem, we let $\gamma = 1$ and $x_3 = 2.5$. We superimpose on the top of the stabilizing region shown (left) for the real plant, the stabilizing sets for the complex plant families $\mathbf{P}_c(\jmath\omega, \theta)$ for $\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi$ are plotted (right).





Example (continue...)

To verify, a number of points inside the performance region, construct the corresponding controllers, and Nyquist plots of $W(s)T(s)$ have been plotted as shown. These points are shown as "*" in (right). We observe from the Nyquist plots and every test set satisfies the H_∞ performance requirement.



Example of FO Controller Design

Consider an unstable plant with time delay.

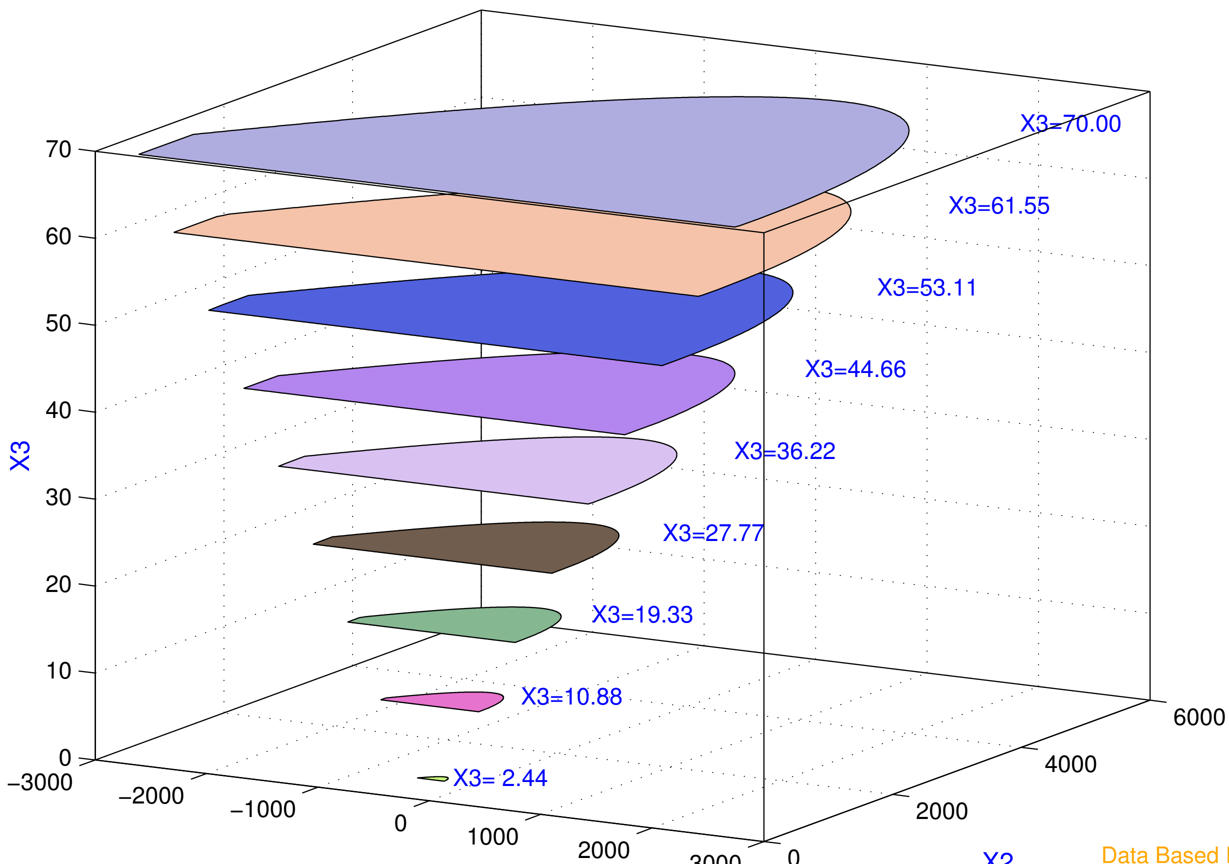
$$G(s) = \left[\frac{s+1}{s^4 + 8s^3 + 48s^2 + 46s - 1}\right] e^{-s}$$

The design specifications are given as follows:

- the closed-loop system must satisfy the following gain and phase margin requirements:

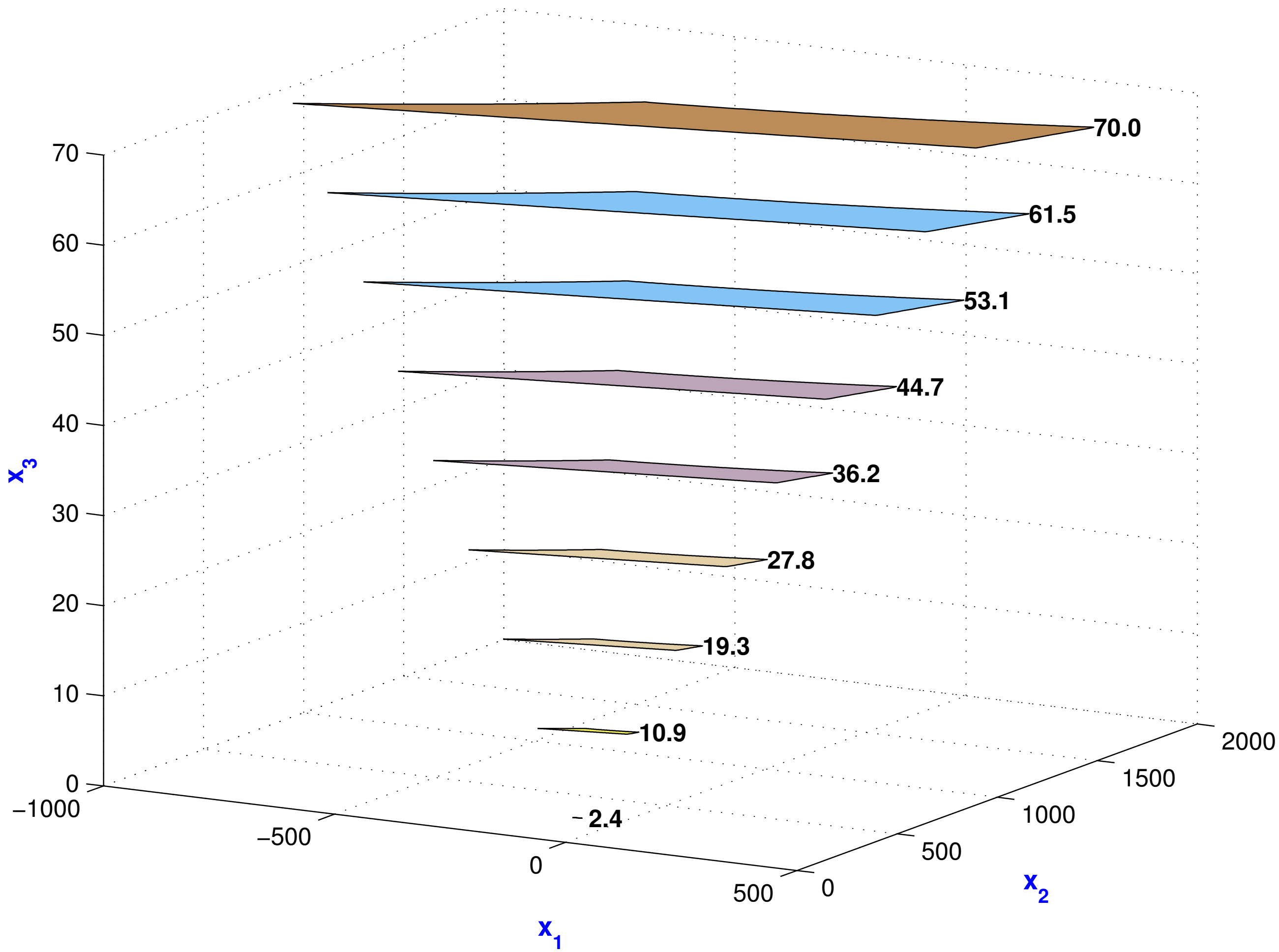
(A) The Complete Set of Stabilizing FO Controllers

The feasible range is found as: $x_3 \in [-6, \infty]$.
We chose to execute the algorithm for $x_3 \in [-6, 70]$.
Then we obtain the the set of FO Controller parameters so that the each and every corresponding closed-loop system satisfies the given gain and phase margin requirements.

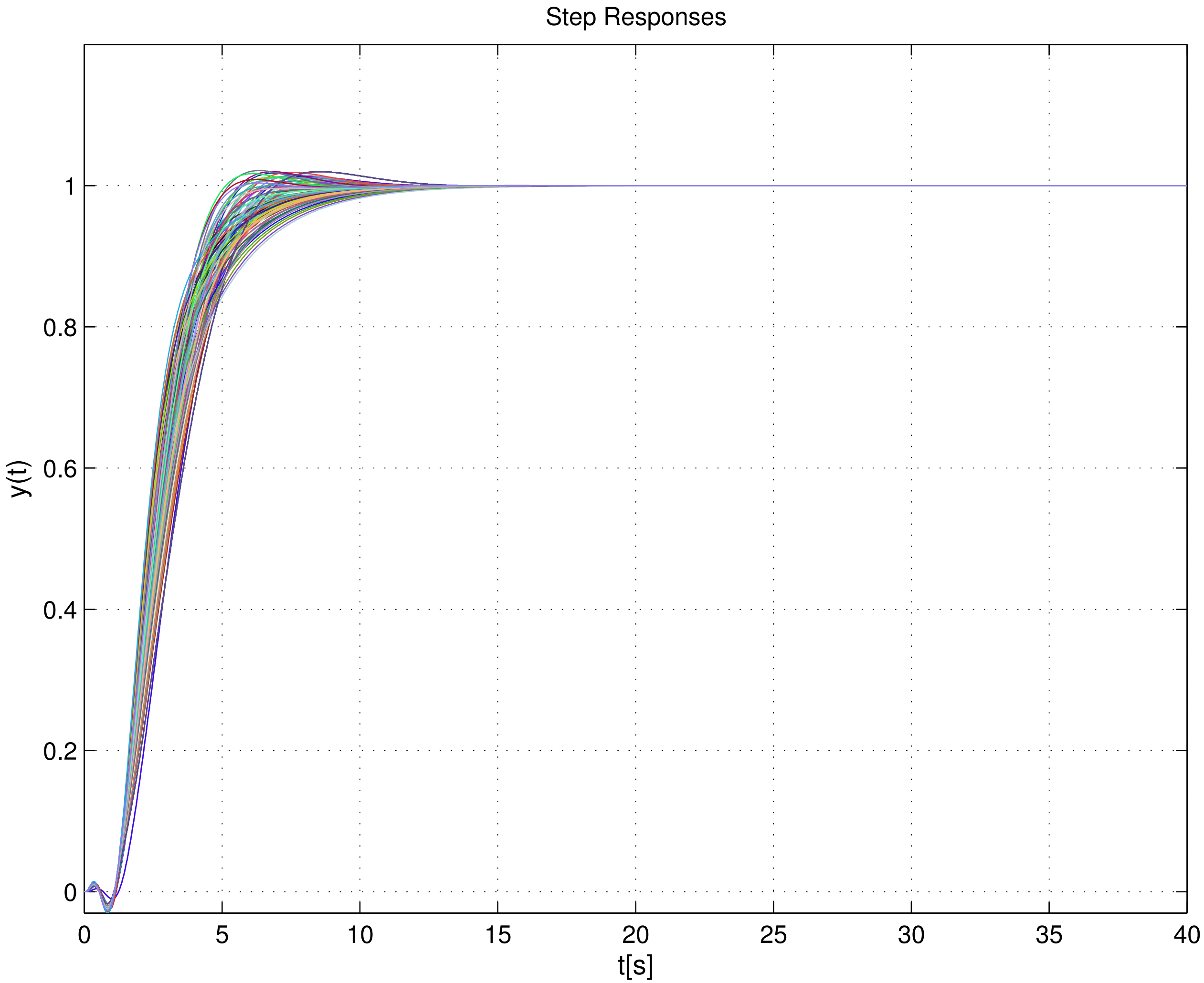


(B) A Set of FOC that Meets the Desired Overshoot and Settling Time Requirements

Computing a feasible range of the generalized time constant based on the CRA, the feasible range of the time constant is obtained as: $\tau \in [2.692,\ 6.258]$.

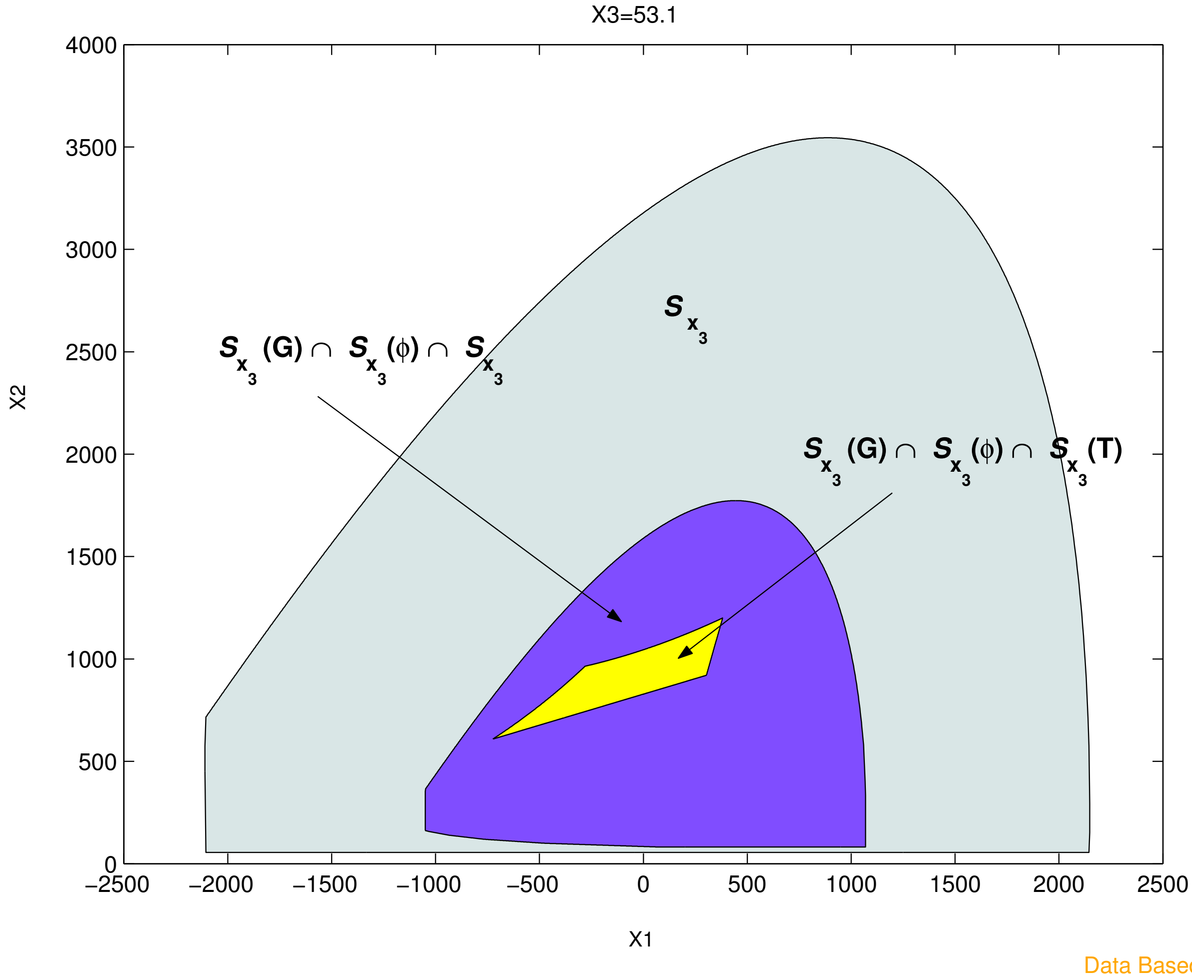


Step responses with FO Controllers in S^*



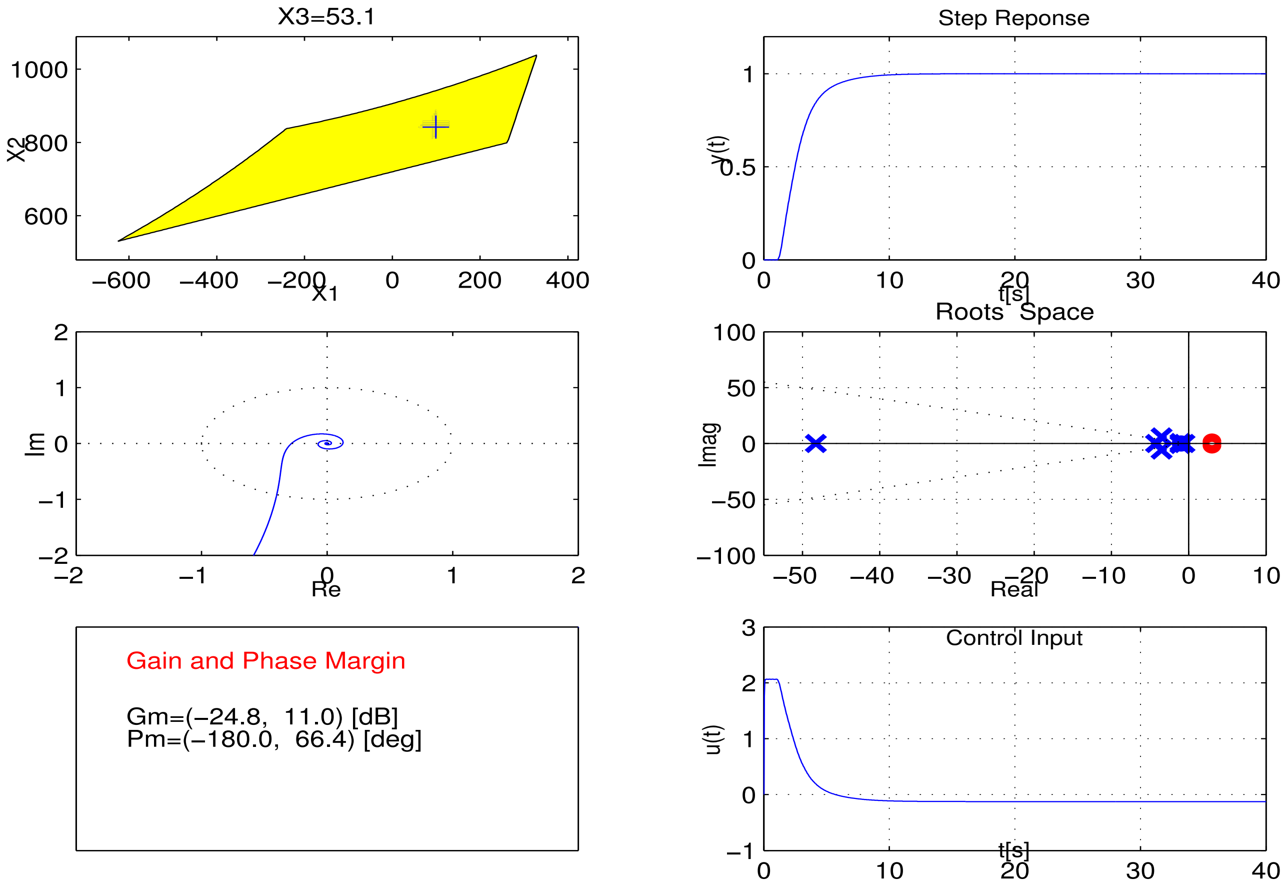
2-D set of FO Controllers, $S^*_{x_3}$, when $x_3 = 53.1$

To proceed, we pick a FO Controller from the controller set S^* and examine various performance of the corresponding closed-loop system. In this example, we first select $x_3 = 53.1$ and the corresponding 2-D set is depicted as in figure.



Various characteristics of the CL system with the selected FOC

A selection of the controller point from the set $\mathcal{S}^*_{x_3}$ with the fixed x_3 enables us to display the following six figures displaying various characteristics of the closed-loop system with the FO Controller chosen.



- The top left figure shows the controller set $\mathcal{S}^*_{x_3}$ and “+” indicates the selected controller of the parameter values:

$$x_1 = 83.3, \quad x_2 = 831, \quad x_3 = 53.1.$$

- The top left figure shows the controller set $\mathcal{S}^*_{x_3}$ and "+" indicates the selected controller of the parameter values:

$$x_1 = 83.3, \quad x_2 = 831, \quad x_3 = 53.1.$$

- The top right figure shows the step response of the closed-loop system. Two figures in the middle show the Nyquist plot, and the closed-loop poles and zeros. The bottom two figures show the values of gain and phase margins, and the control input signal, respectively.

Data-Robust Design (An Example)

What we know:

- $P(j\omega) \pm 20\%$ for all ω

Data-Robust Design (An Example)

What we know:

- $P(j\omega) \pm 20\%$ for all ω
- Number of RHP zeros is $z^{+} = 2$

Data-Robust Design (An Example)

What we know:

- $P(j\omega) \pm 20\%$ for all ω
- Number of RHP zeros is $z^+ = 2$
- Relative degree is $r_P = 1$ due to high frequency slope (-20db/decade)

Data-Robust Design (An Example)

What we know:

- $P(j\omega) \pm 20\%$ for all ω
- Number of RHP zeros is $z^+ = 2$
- Relative degree is $r_P = 1$ due to high frequency slope (-20db/decade)
- Signature required is $r_P + 2z^+ + 2 = 1 + 2(2) + 2 = 7$.

Data-Robust Design (An Example)

What we know:

- $P(j\omega) \pm 20\%$ for all ω
- Number of RHP zeros is $z^+ = 2$
- Relative degree is $r_P = 1$ due to high frequency slope (-20db/decade)
- Signature required is $r_P + 2z^+ + 2 = 1 + 2(2) + 2 = 7$.
- It requires at least 3 frequencies to satisfy $K_p = g(\omega)$.

Notations:

$$\begin{aligned}
\mathbf{P}(j\omega) &= \text{Family of response shown in Bode plot} \\
P_r^{\max}(\omega) &= \max_{P(j\omega)\in\mathbf{P}(j\omega)} P_r(\omega), \quad \text{for every } \omega \\
P_r^{\min}(\omega) &= \min_{P(j\omega)\in\mathbf{P}(j\omega)} P_r(\omega), \quad \text{for every } \omega \\
P_i^{\max}(\omega) &= \max_{P(j\omega)\in\mathbf{P}(j\omega)} P_i(\omega), \quad \text{for every } \omega \\
P_i^{\min}(\omega) &= \min_{P(j\omega)\in\mathbf{P}(j\omega)} P_i(\omega), \quad \text{for every } \omega
\end{aligned}$$

Data-Robust Design (An Example)

What we know:

- $P(j\omega) \pm 20\%$ for all ω
- Number of RHP zeros is $z^+ = 2$
- Relative degree is $r_P = 1$ due to high frequency slope (-20db/decade)
- Signature required is $r_P + 2z^+ + 2 = 1 + 2(2) + 2 = 7$.
- It requires at least 3 frequencies to satisfy $K_p = g(\omega)$.

Notations:

$$
\begin{aligned}
\mathbf{P}(j\omega) &= \text{Family of response shown in Bode plot} \\
P_r^{\max}(\omega) &= \max_{P(j\omega)\in\mathbf{P}(j\omega)} P_r(\omega), \quad \text{for every } \omega \\
P_r^{\min}(\omega) &= \min_{P(j\omega)\in\mathbf{P}(j\omega)} P_r(\omega), \quad \text{for every } \omega \\
P_i^{\max}(\omega) &= \max_{P(j\omega)\in\mathbf{P}(j\omega)} P_i(\omega), \quad \text{for every } \omega \\
P_i^{\min}(\omega) &= \min_{P(j\omega)\in\mathbf{P}(j\omega)} P_i(\omega), \quad \text{for every } \omega
\end{aligned}
$$

Data-Robust Design (An Example) Continue ...

Data-Robust Design (An Example) Continue ...

$$K^* = -\frac{P_r(\omega) + \omega T P_i(\omega)}{|P(j\omega)|^2} := g(\omega)$$

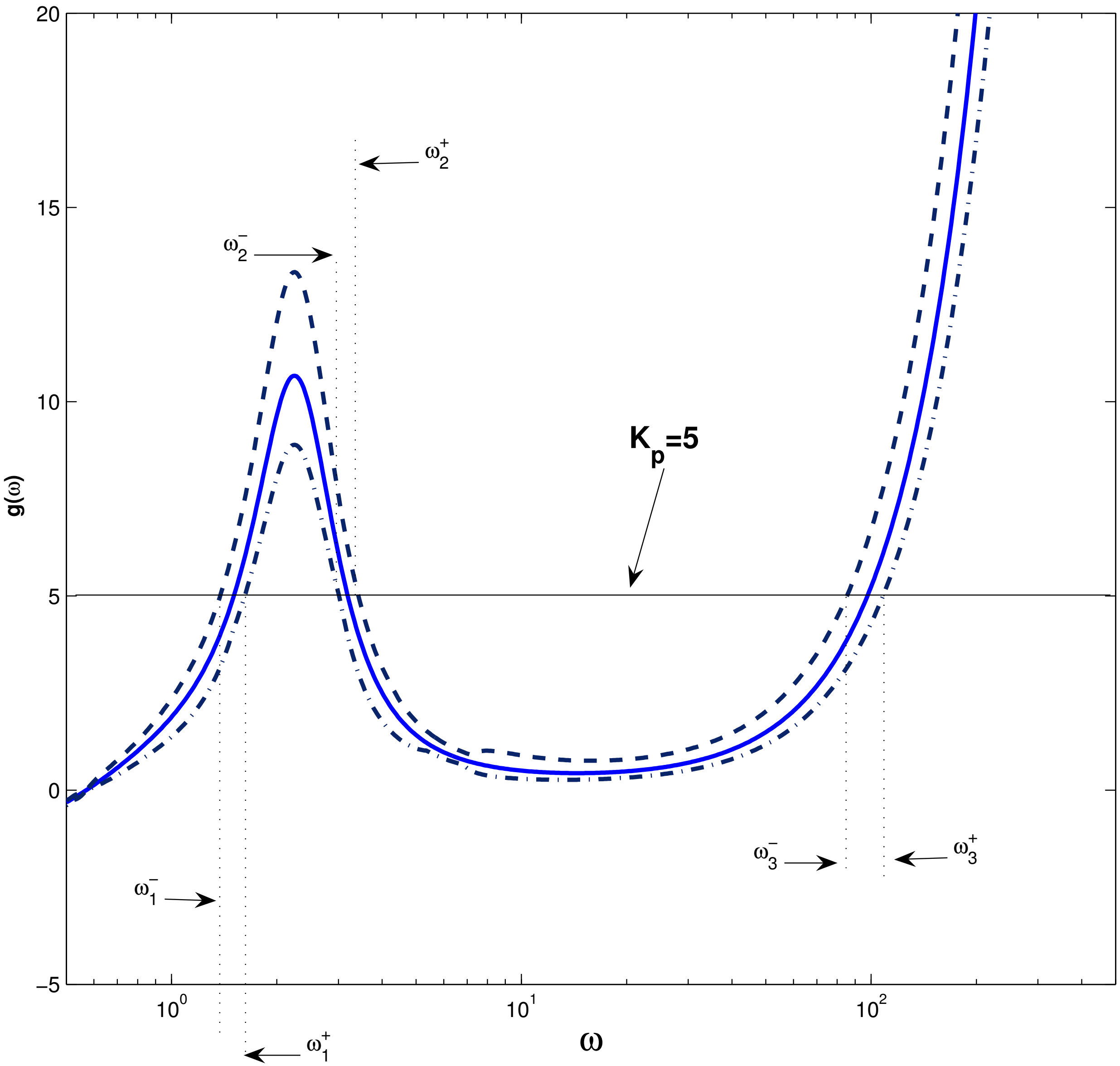
Data-Robust Design (An Example) Continue ...

$$K^* = -\frac{P_r(\omega) + \omega T P_i(\omega)}{|P(j\omega)|^2} := g(\omega)$$

We overbound the $g(\omega)$ family:

$$\begin{aligned} g^{\max}(\omega) &= \max\left\{ g(\omega) \; : \; \begin{array}{c} P_r^{\max}(\omega), P_r^{\min}(\omega), P_i^{\max}(\omega), P_i^{\min}(\omega), \\ |P(j\omega)|^{\max}, |P(j\omega)|^{\min} \end{array} \right\} \\ g^{\min}(\omega) &= \min\left\{ g(\omega) \; : \; \begin{array}{c} P_r^{\max}(\omega), P_r^{\min}(\omega), P_i^{\max}(\omega), P_i^{\min}(\omega), \\ |P(j\omega)|^{\max}, |P(j\omega)|^{\min} \end{array} \right\} \end{aligned}$$

$g(\omega)$ graph: $K_p = 5$

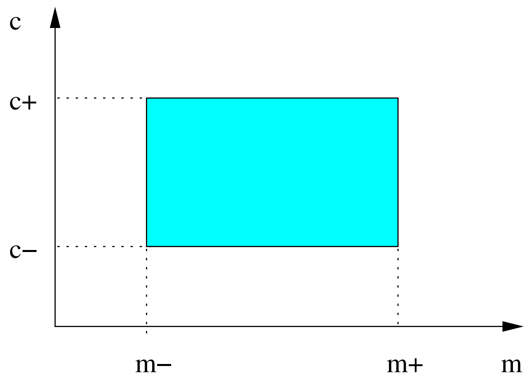


Interval Linear Programming

$$y - \mathbf{m}x - \mathbf{c} > 0, \quad \mathbf{m} \in [m^-, m^+], \;\; \mathbf{c} \in [c^-, c^+].$$

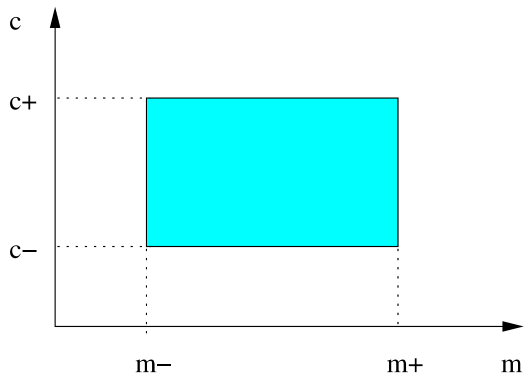
Interval Linear Programming

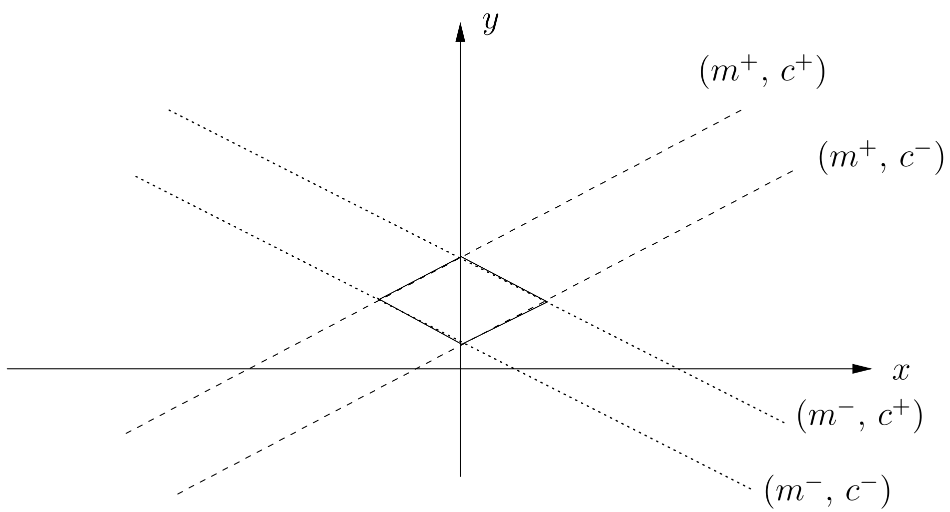
$$y - \mathbf{m}x - \mathbf{c} > 0, \quad \mathbf{m} \in [m^-, m^+], \;\; \mathbf{c} \in [c^-, c^+].$$



Interval Linear Programming

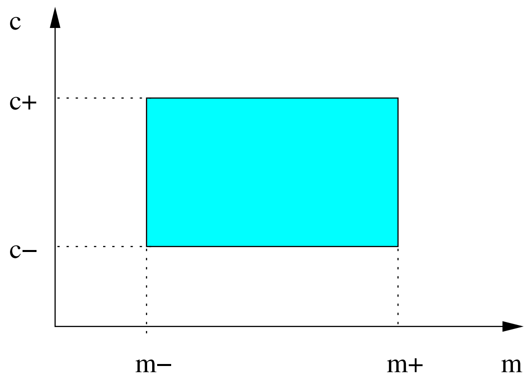
$$y - \mathbf{m}x - \mathbf{c} > 0, \quad \mathbf{m} \in [m^-, m^+], \;\; \mathbf{c} \in [c^-, c^+].$$

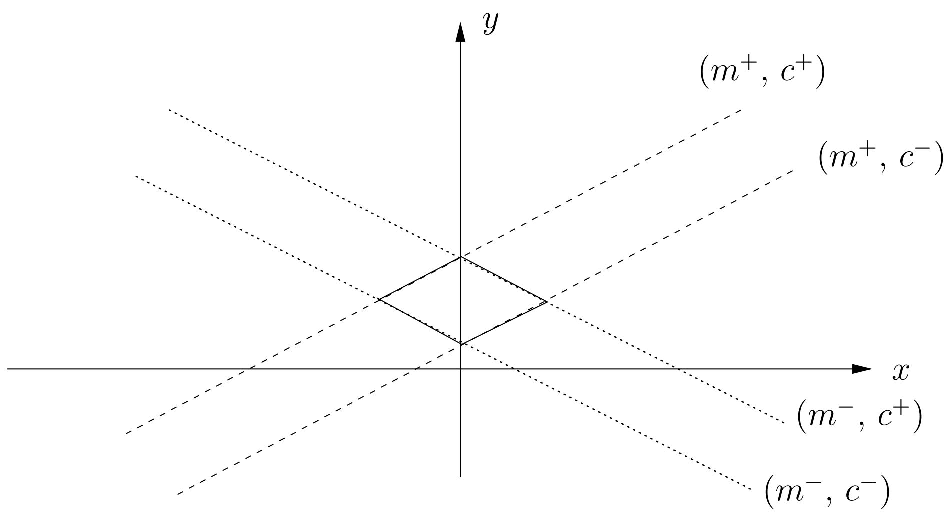




Interval Linear Programming

$$y - \mathbf{m}x - \mathbf{c} > 0, \quad \mathbf{m} \in [m^-, m^+], \;\; \mathbf{c} \in [c^-, c^+].$$





$$\begin{cases} y - m^- x - c^- & > & 0 \\ y - m^- x - c^+ & > & 0 \\ y - m^+ x - c^- & > & 0 \\ y - m^+ x - c^+ & > & 0 \end{cases}$$

A set of linear programmings

- $$b_t = \frac{-\omega_t P_i(\omega_t) + \omega_t^2 T P_r(\omega_t)}{|P(j\omega_t)|^2}$$

A set of linear programmings

- $$b_t = \frac{-\omega_t P_i(\omega_t) + \omega_t^2 T P_r(\omega_t)}{|P(j\omega_t)|^2}$$

- $$\begin{cases} K_i - (\omega_1^-)^2 K_d - b_1^- > 0 \\ K_i - (\omega_2^-)^2 K_d - b_2^- < 0 \\ K_i - (\omega_3^-)^2 K_d - b_3^- > 0 \\ K_i - (\omega_4^-)^2 K_d - b_4^- < 0 \end{cases}$$

A set of linear programmings

- $$b_t = \frac{-\omega_t P_i(\omega_t) + \omega_t^2 T P_r(\omega_t)}{|P(j\omega_t)|^2}$$

- $$\left\{\begin{array}{l} K_i - (\omega_1^-)^2 K_d - b_1^- > 0 \\ K_i - (\omega_2^-)^2 K_d - b_2^- < 0 \\ K_i - (\omega_3^-)^2 K_d - b_3^- > 0 \\ K_i - (\omega_4^-)^2 K_d - b_4^- < 0 \end{array}\right. \qquad \left\{\begin{array}{l} K_i - (\omega_1^-)^2 K_d - b_1^+ > 0 \\ K_i - (\omega_2^-)^2 K_d - b_2^+ < 0 \\ K_i - (\omega_3^-)^2 K_d - b_3^+ > 0 \\ K_i - (\omega_4^-)^2 K_d - b_4^+ < 0 \end{array}\right.$$

A set of linear programmings

- $$b_t = \frac{-\omega_t P_i(\omega_t) + \omega_t^2 T P_r(\omega_t)}{|P(j\omega_t)|^2}$$

- $$\left\{ \begin{array}{l} K_i - (\omega_1^-)^2 K_d - b_1^- > 0 \\ K_i - (\omega_2^-)^2 K_d - b_2^- < 0 \\ K_i - (\omega_3^-)^2 K_d - b_3^- > 0 \\ K_i - (\omega_4^-)^2 K_d - b_4^- < 0 \end{array} \right. \quad \left\{ \begin{array}{l} K_i - (\omega_1^-)^2 K_d - b_1^+ > 0 \\ K_i - (\omega_2^-)^2 K_d - b_2^+ < 0 \\ K_i - (\omega_3^-)^2 K_d - b_3^+ > 0 \\ K_i - (\omega_4^-)^2 K_d - b_4^+ < 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} K_i - (\omega_1^+)^2 K_d - b_1^- > 0 \\ K_i - (\omega_2^+)^2 K_d - b_2^- < 0 \\ K_i - (\omega_3^+)^2 K_d - b_3^- > 0 \\ K_i - (\omega_4^+)^2 K_d - b_4^- < 0 \end{array} \right.$$

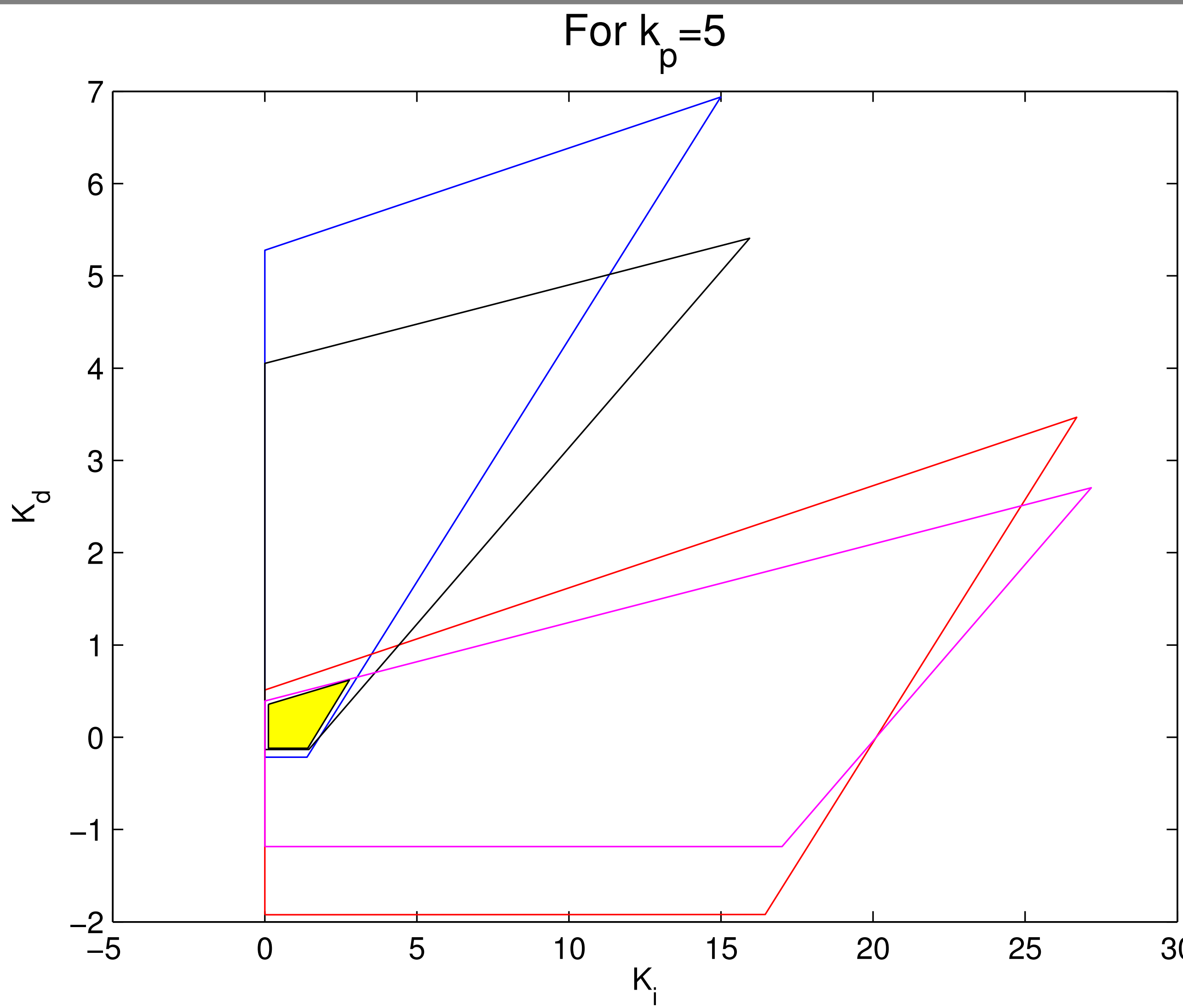
A set of linear programmings

- $$b_t = \frac{-\omega_t P_i(\omega_t) + \omega_t^2 T P_r(\omega_t)}{|P(j\omega_t)|^2}$$

- $$\left\{\begin{array}{l} K_i - (\omega_1^-)^2 K_d - b_1^- > 0 \\ K_i - (\omega_2^-)^2 K_d - b_2^- < 0 \\ K_i - (\omega_3^-)^2 K_d - b_3^- > 0 \\ K_i - (\omega_4^-)^2 K_d - b_4^- < 0 \end{array}\right. \qquad \left\{\begin{array}{l} K_i - (\omega_1^-)^2 K_d - b_1^+ > 0 \\ K_i - (\omega_2^-)^2 K_d - b_2^+ < 0 \\ K_i - (\omega_3^-)^2 K_d - b_3^+ > 0 \\ K_i - (\omega_4^-)^2 K_d - b_4^+ < 0 \end{array}\right.$$

$$\left\{\begin{array}{l} K_i - (\omega_1^+)^2 K_d - b_1^- > 0 \\ K_i - (\omega_2^+)^2 K_d - b_2^- < 0 \\ K_i - (\omega_3^+)^2 K_d - b_3^- > 0 \\ K_i - (\omega_4^+)^2 K_d - b_4^- < 0 \end{array}\right. \qquad \left\{\begin{array}{l} K_i - (\omega_1^+)^2 K_d - b_1^+ > 0 \\ K_i - (\omega_2^+)^2 K_d - b_2^+ < 0 \\ K_i - (\omega_3^+)^2 K_d - b_3^+ > 0 \\ K_i - (\omega_4^+)^2 K_d - b_4^+ < 0 \end{array}\right.$$

Stabilizing set for $K_p = 5$



Digital Control Design

Digital Control Design: Objectives

- To synthesize the set of all stabilizing PID controllers from experimental data of the discrete-time plant rather than from mathematical models.

Digital Control Design: Objectives

- To synthesize the set of all stabilizing PID controllers from experimental data of the discrete-time plant rather than from mathematical models.
- To synthesize the set of all stabilizing First Order controllers from experimental data of the discrete-time plant rather than from mathematical models.

Preliminaries: Tchebyshev Decomposition

Consider a real polynomial in z,

$$P(z) = a_n z^n + a_{n-1} z^{n-1} + a_{n-2} z^{n-2} + \cdots + a_1 z + a_0.$$

Preliminaries: Tchebyshev Decomposition

Consider a real polynomial in z,

$$P(z) = a_n z^n + a_{n-1} z^{n-1} + a_{n-2} z^{n-2} + \cdots + a_1 z + a_0.$$

Since

$$z^k \big|_{z=e^{j\theta}} = \cos k\theta + j \sin k\theta,$$

Preliminaries: Tchebyshev Decomposition

Consider a real polynomial in z,

$$P(z) = a_n z^n + a_{n-1} z^{n-1} + a_{n-2} z^{n-2} + \cdots + a_1 z + a_0.$$

Since

$$z^k \Big|_{z=e^{j\theta}} = \cos k\theta + j \sin k\theta,$$

we have

$$P(e^{j\theta}) = (a_n \cos n\theta + \cdots + a_1 \cos\theta + a_0) + j\,(a_n \sin n\theta + \cdots + a_1 \sin\theta)$$

Preliminaries: Tchebyshev Decomposition

Consider a real polynomial in z,

$$P(z) = a_n z^n + a_{n-1} z^{n-1} + a_{n-2} z^{n-2} + \cdots + a_1 z + a_0.$$

Since

$$z^k \big|_{z=e^{j\theta}} = \cos k\theta + j \sin k\theta,$$

we have

$$P(e^{j\theta}) = (a_n \cos n\theta + \cdots + a_1 \cos\theta + a_0) + j\,(a_n \sin n\theta + \cdots + a_1 \sin\theta)$$

Let

$$u = -\cos\theta,$$

then we have

$$e^{j\theta} = -u + j\sqrt{1-u^2}.$$

Tchebyshev Decomposition (continue ...)

We now have

$$P(e^{j\theta})|_{u=-\cos\theta} = R(u) + j\sqrt{1-u^2}T(u)$$

Tchebyshev Decomposition (continue ...)

We now have

$$P(e^{j\theta})|_{u=-\cos\theta} = R(u) + j\sqrt{1-u^2}T(u)$$

where

$$\begin{aligned} R(u) &= a_n c_n(u) + a_{n-1}c_{n-1}(u) + \cdots + a_1 c_1(u) + a_0 \\ T(u) &= a_n s_n(u) + a_{n-1}s_{n-1}(u) + \cdots + a_1 s_1(u) \end{aligned}$$

Tchebyshev Decomposition (continue ...)

We now have

$$P(e^{j\theta})|_{u=-\cos\theta} = R(u) + j\sqrt{1-u^2}T(u)$$

where

$$\begin{aligned} R(u) &= a_n c_n(u) + a_{n-1}c_{n-1}(u) + \cdots + a_1 c_1(u) + a_0 \\ T(u) &= a_n s_n(u) + a_{n-1}s_{n-1}(u) + \cdots + a_1 s_1(u) \end{aligned}$$

k	$c_k(u)$	$s_k(u)$
1	$-u$	1
2	$2u^2-1$	$-2u$
3	$-4u^3+3u$	$4u^2-1$
4	$8u^4-8u^2+1$	$-8u^3+4u$
5	$-16u^5+20u^3-5u$	$16u^4-12u^2+1$

Tchebyshev Decomposition (continue ...)

We now have

$$P(e^{j\theta})|_{u=-\cos\theta} = R(u) + j\sqrt{1-u^2}T(u)$$

where

$$\begin{aligned} R(u) &= a_n c_n(u) + a_{n-1}c_{n-1}(u) + \cdots + a_1 c_1(u) + a_0 \\ T(u) &= a_n s_n(u) + a_{n-1}s_{n-1}(u) + \cdots + a_1 s_1(u) \end{aligned}$$

k	$c_k(u)$	$s_k(u)$
1	$-u$	1
2	$2u^2 - 1$	$-2u$
3	$-4u^3 + 3u$	$4u^2 - 1$
4	$8u^4 - 8u^2 + 1$	$-8u^3 + 4u$
5	$-16u^5 + 20u^3 - 5u$	$16u^4 - 12u^2 + 1$

$$s_k(u) = -\frac{1}{K}\cdot\frac{dc_k(u)}{du}, \qquad c_{k+1}(u) = -uc_k(u) - \left(1-u^2\right)s_k(u), \quad k = 1, 2, \cdots$$

Preliminaries (continue ...)

Consider a real rational function

$$Q(z) = \frac{P_1(z)}{P_2(z)}$$

where $P_1(z)$ and $P_2(z)$ are polynomials of real coefficients with degrees m and n, respectively. Assume that $P_1(z)$ and $P_2(z)$ have no zeros on the unit circle.

Preliminaries (continue ...)

Consider a real rational function

$$Q(z) = \frac{P_1(z)}{P_2(z)}$$

where $P_1(z)$ and $P_2(z)$ are polynomials of real coefficients with degrees m and n, respectively. Assume that $P_1(z)$ and $P_2(z)$ have no zeros on the unit circle. Write

$$Q(z)\big|_{z=-u+j\sqrt{1-u^2}} = R_q(u) + j\sqrt{1-u^2}T_q(u)$$

Preliminaries (continue ...)

Consider a real rational function

$$Q(z) = \frac{P_1(z)}{P_2(z)}$$

where $P_1(z)$ and $P_2(z)$ are polynomials of real coefficients with degrees m and n, respectively. Assume that $P_1(z)$ and $P_2(z)$ have no zeros on the unit circle.

Write

$$Q(z)\big|_{z=-u+j\sqrt{1-u^2}} = R_q(u) + j\sqrt{1-u^2}\,T_q(u)$$

where

$$R_q(u) = \frac{R(u)}{D(u)}, \quad T_q(u) = \frac{T(u)}{D(u)}$$

Preliminaries (continue ...)

Consider a real rational function

$$Q(z) = \frac{P_1(z)}{P_2(z)}$$

where $P_1(z)$ and $P_2(z)$ are polynomials of real coefficients with degrees m and n, respectively. Assume that $P_1(z)$ and $P_2(z)$ have no zeros on the unit circle.

Write

$$Q(z)\big|_{z=-u+j\sqrt{1-u^2}} = R_q(u) + j\sqrt{1-u^2}T_q(u)$$

where

$$R_q(u) = \frac{R(u)}{D(u)}, \quad T_q(u) = \frac{T(u)}{D(u)}$$

$$\begin{aligned} R(u) &= R_1(u)R_2(u) + \left(1-u^2\right)T_1(u)T_2(u) \\ T(u) &= T_1(u)R_2(u) - R_1(u)T_2(u) \\ D(u) &= R_2^2(u) + \left(1-u^2\right)T_2^2(u) \end{aligned}$$

Preliminaries (continue ...)

Recall

$$Q(z)|_{z=-u+j\sqrt{1-u^2}} = \frac{R(u)}{D(u)} + j\sqrt{1-u^2}\,\frac{T(u)}{D(u)}$$

Preliminaries (continue ...)

Recall

$$Q(z)|_{z=-u+j\sqrt{1-u^2}} = \frac{R(u)}{D(u)} + j\sqrt{1-u^2}\frac{T(u)}{D(u)}$$

Since

$$D(u) = R_2^2(u) + \left(1-u^2\right) T_2^2(u) > 0 \quad \text{for all } u \in [-1, 1],$$

the zeros of $T_q(u)$ are identical to those of $T(u)$.

Preliminaries (continue ...)

Recall

$$Q(z)|_{z=-u+j\sqrt{1-u^2}} = \frac{R(u)}{D(u)} + j\sqrt{1-u^2}\frac{T(u)}{D(u)}$$

Since

$$D(u) = R_2^2(u) + \left(1-u^2\right) T_2^2(u) > 0 \quad \text{for all } u \in [-1,1],$$

the zeros of $T_q(u)$ are identical to those of $T(u)$.

Let $t_1, \cdots, t_k$ denote the real distinct zeros of $T(u)$ of odd multiplicity ordered as

$$-1 < t_1 < t_2 < \cdots < t_k < +1.$$

Schur Signature Lemma

Let $Q(z)$ be a real rational function with i_z zeros and i_p poles, respectively, inside the unit circle $\mathcal{C}$ and no zeros/poles on the unit circle and

$$Q(z)\big|_{z=-u+\sqrt{1-u^2}} = R(u) + j\sqrt{1-u^2}T(u).$$

Schur Signature Lemma

Let $Q(z)$ be a real rational function with i_z zeros and i_p poles, respectively, inside the unit circle $\mathcal{C}$ and no zeros/poles on the unit circle and

$$Q(z)|_{z=-u+\sqrt{1-u^2}} = R(u) + j\sqrt{1-u^2}T(u).$$

Then the net change in phase of $Q(e^{j\theta})$ as θ runs from 0 to π:

$$\Delta_0^{\pi}\angle Q(e^{j\theta}) = \pi\left(i_z - i_p\right) = \pi\sigma[Q(z)]$$

Schur Signature Lemma

Let $Q(z)$ be a real rational function with i_z zeros and i_p poles, respectively, inside the unit circle $\mathcal{C}$ and no zeros/poles on the unit circle and

$$Q(z)\big|_{z=-u+\sqrt{1-u^2}} = R(u) + j\sqrt{1-u^2}T(u).$$

Then the net change in phase of $Q(e^{j\theta})$ as θ runs from 0 to π:

$$\Delta_0^{\pi}\angle Q(e^{j\theta}) = \pi\left(i_z - i_p\right) = \pi\sigma[Q(z)]$$

The signature is

$$\begin{aligned}\sigma[Q(z)] &= i_z - i_p \\ &= \frac{1}{2}\text{sgn}\left[T^{(p)}(-1)\right]\left(\text{sgn}[R(-1)] + 2\sum_{j=1}^{k}(-1)^j\text{sgn}[R(t_j)] + (-1)^{k+1}\text{sgn}[R(+1)]\right).\end{aligned}$$

Assumption

- The plant is stabilizable (mathematically, $N(z)$ and $D(z)$ are coprime). where

$$P(z) = \frac{N(z)}{D(z)}$$

Assumption

- The plant is stabilizable (mathematically, $N(z)$ and $D(z)$ are coprime). where

$$P(z) = \frac{N(z)}{D(z)}$$

- The plant has no poles on the unit circle.

Assumption

- The plant is stabilizable (mathematically, $N(z)$ and $D(z)$ are coprime). where

$$P(z) = \frac{N(z)}{D(z)}$$

- The plant has no poles on the unit circle.
- Only available information:

Assumption

- The plant is stabilizable (mathematically, $N(z)$ and $D(z)$ are coprime). where

$$P(z) = \frac{N(z)}{D(z)}$$

- The plant has no poles on the unit circle.
- Only available information:
  - $P(e^{j\theta})$ for $\theta \in [0, 2\pi]$

Assumption

- The plant is stabilizable (mathematically, $N(z)$ and $D(z)$ are coprime). where

$$P(z) = \frac{N(z)}{D(z)}$$

- The plant has no poles on the unit circle.
- Only available information:
  - $P(e^{j\theta})$ for $\theta \in [0, 2\pi]$
  - Number of unstable poles of the plant, o_p

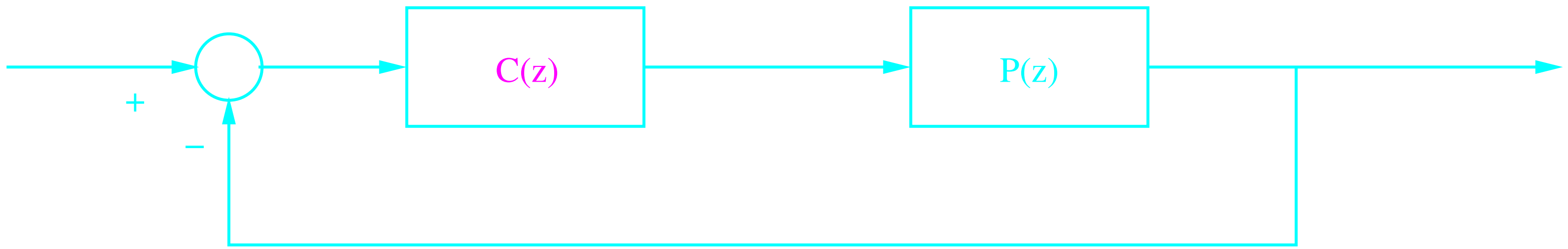
Assumption

- The plant is stabilizable (mathematically, $N(z)$ and $D(z)$ are coprime). where

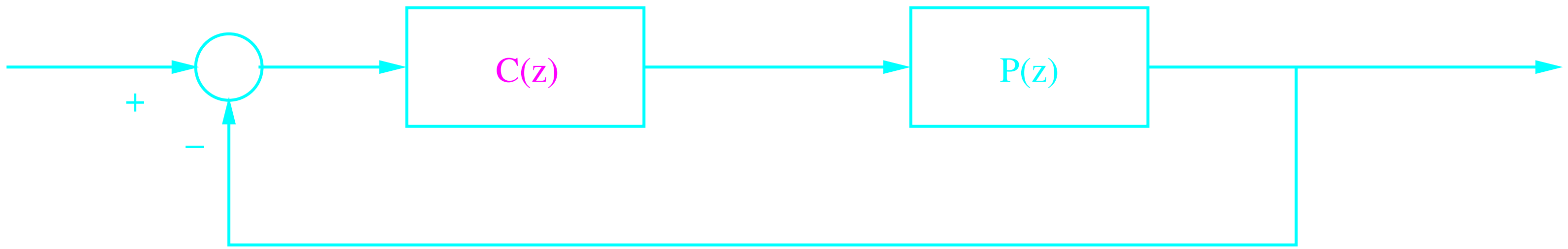
$$P(z) = \frac{N(z)}{D(z)}$$

- The plant has no poles on the unit circle.
- Only available information:
  - $P(e^{j\theta})$ for $\theta \in [0, 2\pi]$
  - Number of unstable poles of the plant, o_p
  - The relative degree r

PID Controller Design for Discrete-time Systems



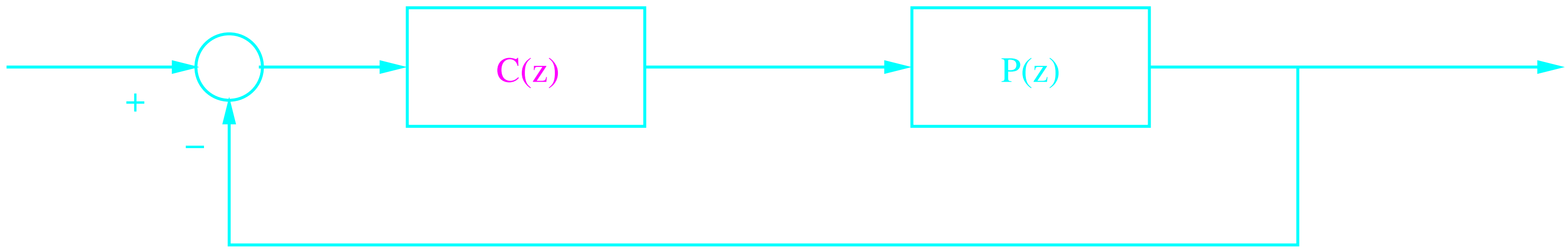
PID Controller Design for Discrete-time Systems



Let the PID controller be of the form

$$C(z) = K_P + \frac{K_I T z}{z-1} + \frac{K_D (z-1)}{Tz}$$

PID Controller Design for Discrete-time Systems



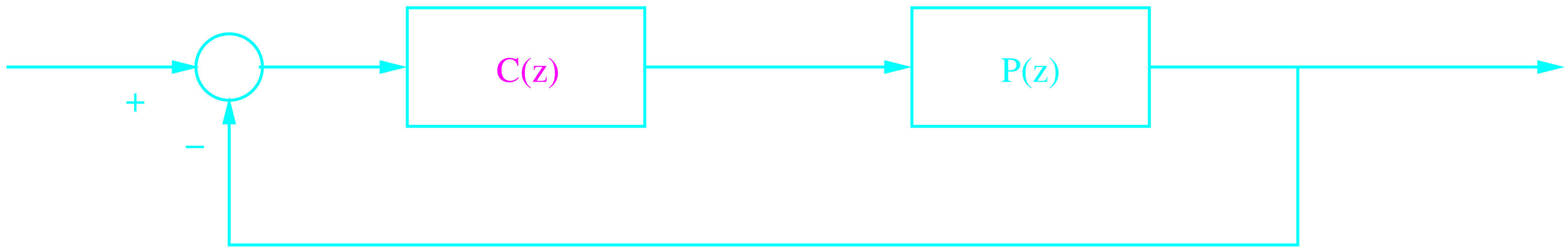
Let the PID controller be of the form

$$C(z) = K_P + \frac{K_I T z}{z-1} + \frac{K_D (z-1)}{Tz}$$

equivalently,

$$C(z) = \frac{K_2 z^2 + K_1 z + K_0}{z(z-1)}$$

PID Controller Design for Discrete-time Systems



Let the PID controller be of the form

$$C(z) = K_P + \frac{K_I T z}{z-1} + \frac{K_D(z-1)}{Tz}$$

equivalently,

$$C(z) = \frac{K_2 z^2 + K_1 z + K_0}{z(z-1)}$$

where

$$K_P = -K_1 - 2K_0, \quad K_I = \frac{K_0 + K_1 + K_2}{T}, \quad K_D = K_0 T$$

PID Controller Design (continue ...)

The closed-loop characteristic polynomial is

$$\delta(z) := z(z-1)D(z) + \left(K_2 z^2 + K_1 z + K_0\right) N(z).$$

PID Controller Design (continue ...)

The closed-loop characteristic polynomial is

$$\delta(z) := z(z-1)D(z) + \left(K_2 z^2 + K_1 z + K_0\right) N(z).$$

Closed-loop stability requires $\sigma(\delta) = n + 2$.

PID Controller Design (continue ...)

The closed-loop characteristic polynomial is

$$\delta(z) := z(z-1)D(z) + \left(K_2 z^2 + K_1 z + K_0\right) N(z).$$

Closed-loop stability requires $\sigma(\delta) = n + 2$.

Consider

$$\Pi(z) = \frac{\delta(z)}{D(z)} = z(z-1) + \left(K_2 z^2 + K_1 z + K_0\right) P(z)$$

PID Controller Design (continue ...)

The closed-loop characteristic polynomial is

$$\delta(z) := z(z-1)D(z) + \left(K_2 z^2 + K_1 z + K_0\right) N(z).$$

Closed-loop stability requires $\sigma(\delta) = n + 2$.

Consider

$$\Pi(z) = \frac{\delta(z)}{D(z)} = z(z-1) + \left(K_2 z^2 + K_1 z + K_0\right) P(z)$$

Consequently, the stability requires that

$$\sigma(\Pi) = n + 2 - i_p$$

where i_p is the number of poles of the plant located inside unit circle.

Stability Condition with PID Controllers

Let $P(z)$ be the plant with relative degree r. Let the PID controller be

$$C(z) = \frac{K_2 z^2 + K_1 z + K_0}{z(z-1)}.$$

Stability Condition with PID Controllers

Let $P(z)$ be the plant with relative degree r. Let the PID controller be

$$C(z) = \frac{K_2 z^2 + K_1 z + K_0}{z(z-1)}.$$

Then the closed-loop system is stable if and only if

$$\sigma(\nu) = r + o_z + 1$$

where

$$\begin{aligned} \nu(z) &= z^{-1} P(z^{-1}) \Pi(z) \\ \Pi(z) &= z(z-1) + \left(K_2 z^2 + K_1 z + K_0\right) P(z) \end{aligned}$$

and o_z is the number of non-minimum phase zeros of $P(z)$.

PID Controller Design (continue ...)

Now

$$\nu(z)|_{z=-u+jv} = z^{-1}P(z^{-1})\Pi(z)\big|_{z=-u+jv}$$
$$= \left[z^{-1}P(z^{-1}) + \left(K_0 z^{-1} + K_1 + K_0 z\right) P(z)P(z^{-1})\right]_{z=-u+jv}$$
$$= (-u-1+jv)\left(R_p(u) - jvT_p(u)\right) + \left[K_0(-u-jv) + K_1 + K_2(-u+jv)\right] m^2(u)$$

where

$$v = \sqrt{1-u^2}, \qquad m(u) = |P(z)|_{z=-u+jv}.$$

PID Controller Design (continue ...)

Now

$$\begin{aligned}\nu(z)|_{z=-u+jv} &= z^{-1}P(z^{-1})\Pi(z)\big|_{z=-u+jv} \\ &= \left[z^{-1}P(z^{-1}) + \left(K_0 z^{-1} + K_1 + K_0 z\right) P(z)P(z^{-1})\right]_{z=-u+jv} \\ &= (-u-1+jv)\left(R_p(u) - jvT_p(u)\right) + \left[K_0(-u-jv) + K_1 + K_2(-u+jv)\right] m^2(u)\end{aligned}$$

where

$$v = \sqrt{1-u^2}, \qquad m(u) = |P(z)|_{z=-u+jv}.$$

Then

$$\nu(z) = R_\nu(u, K_0, K_1, K_3) + jvT_\nu(u, K_3)$$

where

$$K_3 := K_2 - K_0$$

PID Controller Design (continue ...)

$$
\begin{aligned}
R_\nu(u, K_0, K_1, K_3) &= -(u-1)R_p(u) - (1-u^2)T_p(u) + K_1 m^2(u) \\
&\quad - u(2K_0 + K_3)m^2(u) \\
T_\nu(u, K_3) &= R_p(u) - (u+1)T_p(u) + K_3 m^2(u).
\end{aligned}
$$

- Note that

$$
P(z)\big|_{z=-u+j\sqrt{1-u^2}} = R_p(u) + j\sqrt{1-u^2}\,T_p(u)
$$

PID Controller Design (continue ...)

$$\begin{aligned} R_\nu(u, K_0, K_1, K_3) &= -(u-1)R_p(u) - (1-u^2)T_p(u) + K_1 m^2(u) \\ &\quad -u(2K_0 + K_3)m^2(u) \\ T_\nu(u, K_3) &= R_p(u) - (u+1)T_p(u) + K_3 m^2(u). \end{aligned}$$

- Note that
$$P(z)\big|_{z=-u+j\sqrt{1-u^2}} = R_p(u) + j\sqrt{1-u^2}\,T_p(u)$$
- $R_p(u)$ and $T_P(u)$ can be obtained directly from the experimental data $P(e^{j\theta})|_{u=-\cos\theta}$,

PID Controller Design (continue ...)

$$
\begin{aligned}
R_\nu(u, K_0, K_1, K_3) &= -(u-1)R_p(u) - (1-u^2)T_p(u) + K_1 m^2(u) \\
&\quad -u(2K_0 + K_3)m^2(u) \\
T_\nu(u, K_3) &= R_p(u) - (u+1)T_p(u) + K_3 m^2(u).
\end{aligned}
$$

- Note that

$$
P(z)\big|_{z=-u+j\sqrt{1-u^2}} = R_p(u) + j\sqrt{1-u^2}\,T_p(u)
$$

- $R_p(u)$ and $T_P(u)$ can be obtained directly from the experimental data $P(e^{j\theta})|_{u=-\cos\theta}$,
- $R_\nu(u, K_0, K_1, K_3)$ and $T_\nu(u, K_3)$ can also be obtained from $P(e^{j\theta})$

Algorithm: PID Controller Design

- For fixed $K_3 = K_3^*$, solve

$$K_3^* = \frac{-R_p(u) + (u+1)T_p(u)}{m^2(u)} = g(u)$$

determine the roots $u_1, u_2, \cdots$

$$u_0 = -1 < u_1 < u_2 < \cdots < u_l < u_{l+1} = +1$$

Algorithm: PID Controller Design

- For fixed $K_3 = K_3^*$, solve

$$K_3^* = \frac{-R_p(u) + (u+1)T_p(u)}{m^2(u)} = g(u)$$

determine the roots $u_1, u_2, \cdots$

$$u_0 = -1 < u_1 < u_2 < \cdots < u_l < u_{l+1} = +1$$

- Develop linear inequalities corresponding to stability as follows. Let

$$I^j = \left\{ i_0^j, i_1^j, \cdots, i_l^j, i_{l+1}^j \right\}$$

denote a string where $i_i^j \in \{0, 1, -1\}$ such that

$$i_0^j - 2i_1^j + 2i_2^j - \cdots + (-1)^{l+1} i_{l+1}^j = r + o_z + 1.$$

Algorithm: PID Controller Design (continue ...)

- For each string I^j satisfying the above, we have the set of inequalities

$$\operatorname{sgn}\left[R_\nu\left(u_t, K_0, K_1, K_3^*\right)\right] i_t^j > 0$$

which is a set of linear inequalities in K_0, K_1 space for fixed K_3^*.

Algorithm: PID Controller Design (continue ...)

- For each string I^j satisfying the above, we have the set of inequalities

$$\operatorname{sgn}\left[R_\nu\left(u_t, K_0, K_1, K_3^*\right)\right] i_t^j > 0$$

which is a set of linear inequalities in K_0, K_1 space for fixed K_3^*.

- By constructing these inequalities for each string satisfying the above, we obtain the stabilizing set for $K_3 = K_3^*$.

Algorithm: PID Controller Design (continue ...)

- For each string I^j satisfying the above, we have the set of inequalities

$$\operatorname{sgn}\left[R_\nu\left(u_t, K_0, K_1, K_3^*\right)\right] i_t^j > 0$$

which is a set of linear inequalities in K_0, K_1 space for fixed K_3^*.

- By constructing these inequalities for each string satisfying the above, we obtain the stabilizing set for $K_3 = K_3^*$.
- By sweeping over K_3 we can generate the complete set.

Algorithm: PID Controller Design (continue ...)

- For each string I^j satisfying the above, we have the set of inequalities

$$\operatorname{sgn}\left[R_\nu\left(u_t, K_0, K_1, K_3^*\right)\right] i_t^j > 0$$

  which is a set of linear inequalities in K_0, K_1 space for fixed K_3^*.
- By constructing these inequalities for each string satisfying the above, we obtain the stabilizing set for $K_3 = K_3^*$.
- By sweeping over K_3 we can generate the complete set.

- The range of K_3 to be swept is determined by the requirement that $g(u)$ should have $\frac{o_z+1}{2}$ roots at least.

An Example: PID Design

Available Information:

- Frequency domain data

$$\mathbf{P}(e^{\jmath\omega T}) := \left\{ P(e^{\jmath\omega T}), \omega = \frac{2\pi}{T} \text{ sampled every } T = 0.01 \right\}.$$

An Example: PID Design

Available Information:

- Frequency domain data

$$\mathbf{P}(e^{\jmath\omega T}) := \left\{ P(e^{\jmath\omega T}), \omega = \frac{2\pi}{T} \text{ sampled every } T = 0.01 \right\}.$$

- The plant is stable. In other words, the number of unstable poles of the plant is 0, that is, $o_p = 0$.

An Example: PID Design

Available Information:

- Frequency domain data

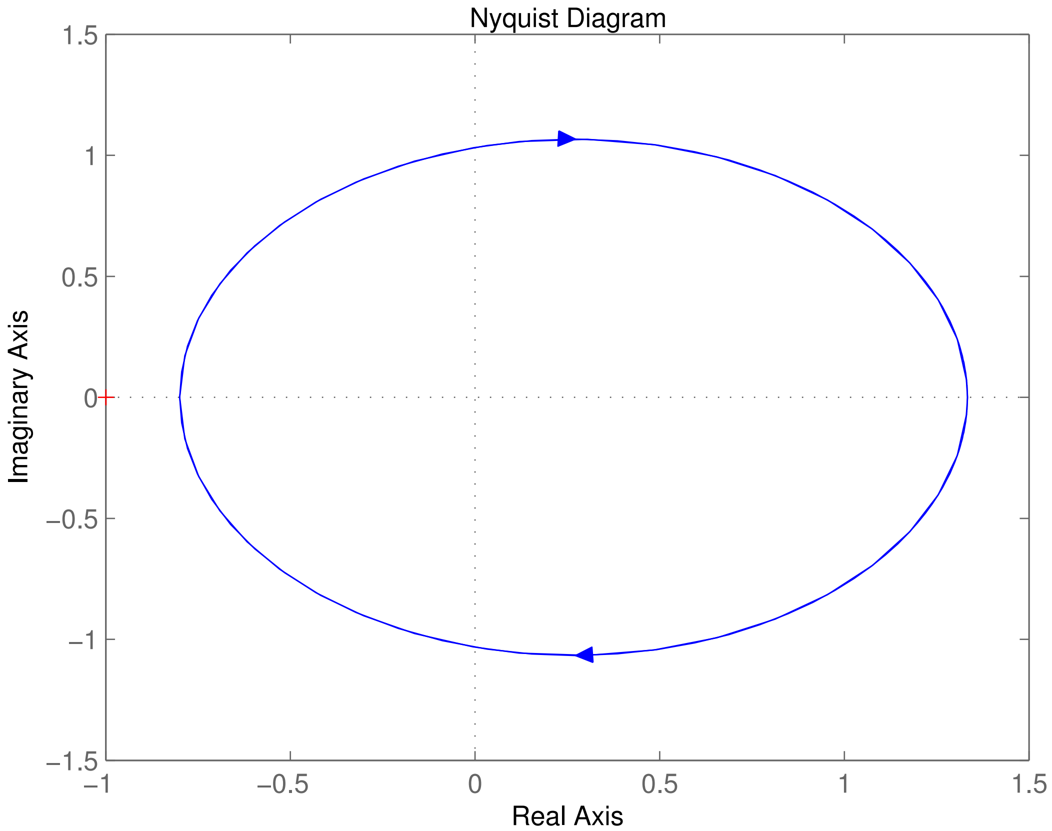
$$\mathbf{P}(e^{\jmath\omega T}) := \left\{ P(e^{\jmath\omega T}), \omega = \frac{2\pi}{T} \text{ sampled every } T = 0.01 \right\}.$$

- The plant is stable. In other words, the number of unstable poles of the plant is 0, that is, $o_p = 0$.

- The relative degree of the plant is 2, that is, $r = 2$.

An Example: PID Design (continue ...)

The Nyquist plot of the plant $P(z)$:



An Example: PID Design (continue ...)

- Net phase:

$$\Delta_0^{\pi} \angle P(e^{j\theta}) = -\pi \left[r + (o_z - o_p) \right] := -2\pi. \quad \Rightarrow \quad o_z = 2 - r + o_p = 2 - 2 + 0 = 0.$$

An Example: PID Design (continue ...)

- Net phase:

$$\Delta_0^{\pi} \angle P(e^{j\theta}) = -\pi \left[r + (o_z - o_p) \right] := -2\pi. \quad \Rightarrow \quad o_z = 2 - r + o_p = 2 - 2 + 0 = 0.$$

- The stability requirement is equivalent to

$$\sigma \left[\nu(z) \right] = 2 + 0 + 1 = 3.$$

An Example: PID Design (continue ...)

- Net phase:

$$\Delta_0^{\pi} \angle P(e^{j\theta}) = -\pi \left[r + (o_z - o_p) \right] := -2\pi. \quad \Rightarrow \quad o_z = 2 - r + o_p = 2 - 2 + 0 = 0.$$

- The stability requirement is equivalent to

$$\sigma \left[\nu(z) \right] = 2 + 0 + 1 = 3.$$

- Applying this to the signature lemma,

$$\frac{1}{2} \mathrm{sgn}[T(-1)] \left(\mathrm{sgn}[R(-1)] - 2\mathrm{sgn}[R(t_1)] + 2\mathrm{sgn}[R(t_2)] - \cdots \mathrm{sgn}[R(1)] \right) := 3$$

where t_i are the zeros of $g(u)$ in $g(u)$ for fixed K_3.

An Example: PID Design (continue ...)

- It is easy to see that at least two zeros t_i are required and also that the only feasible string of sign sequences is:

sgn of	$T(-1)$	$R(-1)$	$R(t_1)$	$R(t_2)$	$R(1)$
	1	1	-1	1	-1

An Example: PID Design (continue ...)

- It is easy to see that at least two zeros t_i are required and also that the only feasible string of sign sequences is:

sgn of	$T(-1)$	$R(-1)$	$R(t_1)$	$R(t_2)$	$R(1)$
	1	1	-1	1	-1

- The feasible range of K_3 values is that corresponding to the requirement of two zeros in $T(u)$.

An Example: PID Design (continue ...)

- It is easy to see that at least two zeros t_i are required and also that the only feasible string of sign sequences is:

sgn of	$T(-1)$	$R(-1)$	$R(t_1)$	$R(t_2)$	$R(1)$
	1	1	-1	1	-1

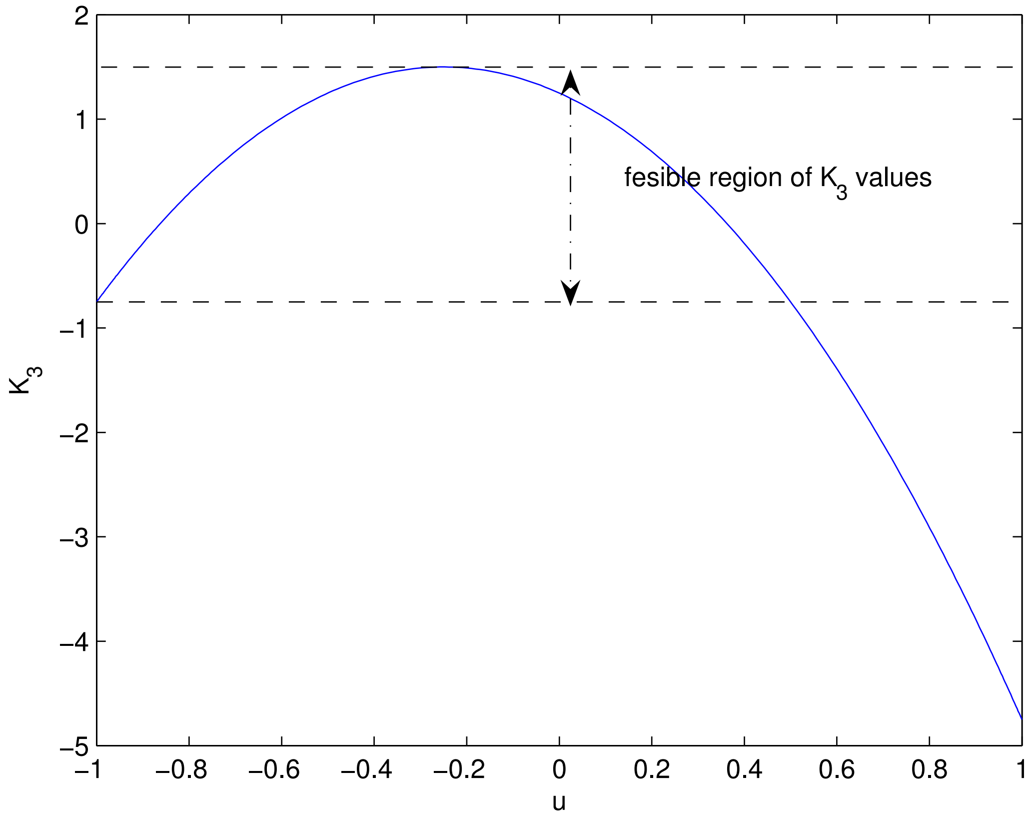
- The feasible range of K_3 values is that corresponding to the requirement of two zeros in $T(u)$.
- Plot the right hand side of $g(u)$:

$$g(u) = \frac{1}{|P_c(u)|^2}\left(-P_r(u) - (1+u)P_i(u)\right) = K_3$$

where

$$P(e^{j\omega T})|_{\omega T=\theta} = P(e^{j\theta})|_{u=-\cos\theta} = P_r(u) + j\sqrt{1-u^2}P_i(u).$$

An Example: PID Design (continue ...)



An Example: PID Design (continue ...)

- At $K_3 = 1.3$, it is found from the graph that

$$u = -0.4736 := t_1, \qquad u = -0.0264 := t_2.$$

An Example: PID Design (continue ...)

- At $K_3 = 1.3$, it is found from the graph that

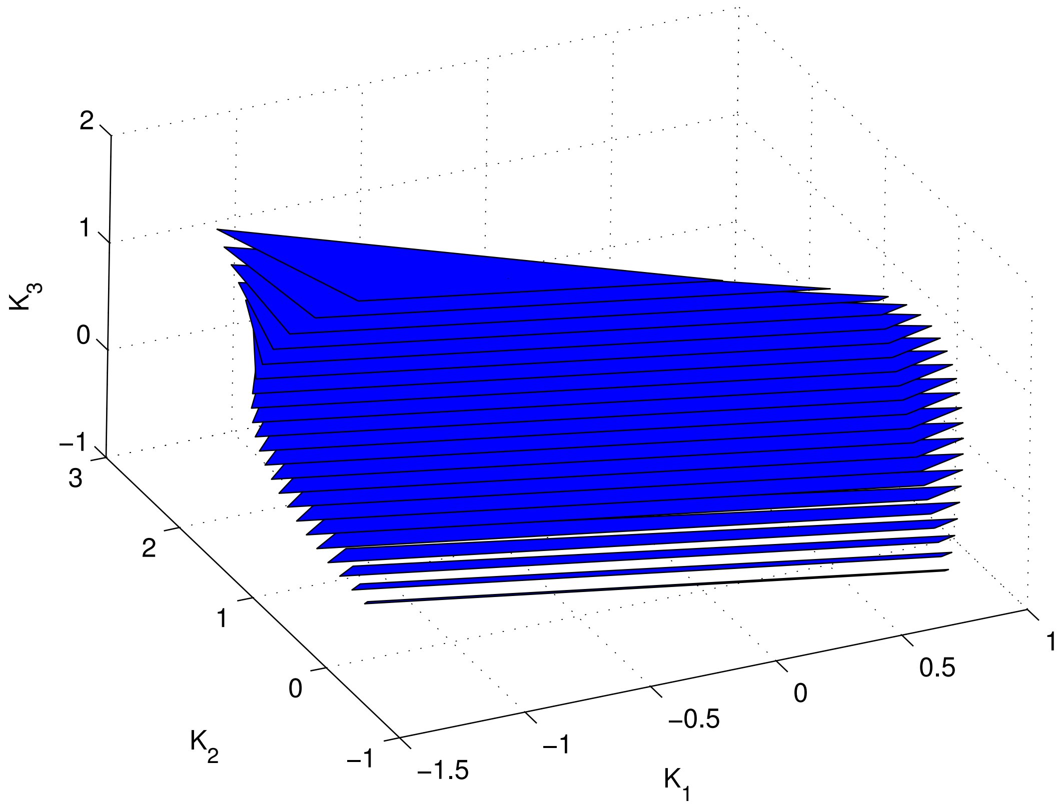
$$u = -0.4736 := t_1, \qquad u = -0.0264 := t_2.$$

- Then the set of linear inequalities corresponding to $K_3 = 1.3$ is

$$\begin{aligned}
T(-1) &= 1 \\
R(-1) &= -2.3111 + 1.7778K_1 + 3.5556K_2 > 0 \\
R(-0.4736) &= -0.6939 + 0.7473K_1 + 0.7078K_2 < 0 \\
R(-0.0264) &= 0.7226 + 0.6403K_1 + 0.0338K_2 > 0 \\
R(1) &= -0.3556 + 1.7778K_1 - 3.5556K_2 < 0
\end{aligned}$$

An Example: PID Design (continue ...)

By sweeping K_3 over $(-0.7, 1.4)$, we have the stabilizing PID parameter regions shown:



1st Order Controllers for Discrete-time Systems

- Consider the frequency response of the discrete-time plant P:

$$P(z)|_{z=-u+jv} = R_p(u) + jvT_p(u).$$

1st Order Controllers for Discrete-time Systems

- Consider the frequency response of the discrete-time plant P:

$$P(z)|_{z=-u+jv} = R_p(u) + jvT_p(u).$$

- Note that $R_P(u)$ and $T_p(u)$ for $-1 \leq u \leq 1$ are immediately available from the given frequency response data points provided by $P(e^{j\theta})$ for $\theta \in [0, \pi]$.

1st Order Controllers for Discrete-time Systems

- Consider the frequency response of the discrete-time plant P:

$$P(z)|_{z=-u+jv} = R_p(u) + jvT_p(u).$$

- Note that $R_P(u)$ and $T_p(u)$ for $-1 \leq u \leq 1$ are immediately available from the given frequency response data points provided by $P(e^{j\theta})$ for $\theta \in [0, \pi]$.

- Consider the real rational function

$$F(z) = (z + x_3) + (zx_1 + x_2)\, P(z).$$

Stability Condition with First Order Controllers

Let $P(z)$ be the plant with the number of unstable poles being o_p. Let the first order controller be

$$C(z) = \frac{x_1 z + x_2}{z + x_3}.$$

Stability Condition with First Order Controllers

Let $P(z)$ be the plant with the number of unstable poles being o_p. Let the first order controller be

$$C(z) = \frac{x_1 z + x_2}{z + x_3}.$$

Then the closed-loop system is stable if and only if

$$\sigma(\Pi) = o_p + 1$$

where

$$\Pi(z) = (z + x_3) + (zx_1 + x_2)\, P(z)$$

and o_p is the number of non-minimum phase poles of $P(z)$.

1st Order Controllers Design

Consider

$$\Pi(z) = (z + x_3) + (zx_1 + x_2)\, P(z)$$

and

$$\begin{aligned}
&\Pi(z)|_{z=-u+jv} \\
= \quad & (-u + x_3 + jv) + \Big(-ux_1 + x_2 + jvx_1\Big)\Big(R_p(u) + jvT_p(u)\Big) \\
= \quad & (x_3 - u) + R_p(u)x_2 - \Big(uR_p(u) + v^2T_p(u)\Big)x_1 \\
& \qquad + jv\left[\Big(R_p(u) - uT_p(u)\Big)x_1 + T_p(u)x_2 + 1\right].
\end{aligned}$$

Stability Conditions

For complex root crossing, we now have the expression of the curve in (x_1, x_2) space for every fixed x_3.

$$\underbrace{\begin{bmatrix} -\left(uR_p(u) + v^2T_p(u)\right) & R_p(u) \\ v\left(R_p(u) - uT_p(u)\right) & vT_p(u) \end{bmatrix}}_{A(u)} \begin{bmatrix} x_1(u) \\ x_2(u) \end{bmatrix} = \begin{bmatrix} -(x_3 - u) \\ -v \end{bmatrix}.$$

Stability Conditions

For complex root crossing, we now have the expression of the curve in (x_1, x_2) space for every fixed x_3.

$$\underbrace{\begin{bmatrix} -\left(uR_p(u) + v^2T_p(u)\right) & R_p(u) \\ v\left(R_p(u) - uT_p(u)\right) & vT_p(u) \end{bmatrix}}_{A(u)} \begin{bmatrix} x_1(u) \\ x_2(u) \end{bmatrix} = \begin{bmatrix} -(x_3 - u) \\ -v \end{bmatrix}.$$

Since

$$\det[A(u)] = -\left(R_p^2(u) + v^2T_p^2(u)\right)^2 v = -v\left|P(e^{j\theta})\right|,$$

Stability Conditions

For complex root crossing, we now have the expression of the curve in (x_1, x_2) space for every fixed x_3.

$$\underbrace{\begin{bmatrix} -\left(uR_p(u) + v^2T_p(u)\right) & R_p(u) \\ v\left(R_p(u) - uT_p(u)\right) & vT_p(u) \end{bmatrix}}_{A(u)} \begin{bmatrix} x_1(u) \\ x_2(u) \end{bmatrix} = \begin{bmatrix} -(x_3 - u) \\ -v \end{bmatrix}.$$

Since

$$\det[A(u)] = -\left(R_p^2(u) + v^2T_p^2(u)\right)^2 v = -v\left|P(e^{j\theta})\right|,$$

the solution of the above is

$$\begin{bmatrix} x_1(u) \\ x_2(u) \end{bmatrix} = -\frac{1}{\left|P(e^{j\theta})\right|^2} \begin{bmatrix} (u - x_3)T_p(u) + R_p(u) \\ (1 - ux_3)T_p(u) + x_3R_p(u) \end{bmatrix}.$$

Stability Conditions (continue ...)

The two straight lines representing the real root crossing can be obtained by letting $u = -1$ and $u = 1$, equivalently letting $\theta = 0$ and $\theta = \pi$.

$$\begin{aligned} (x_3 - 1) + P(e^{\jmath 0})(x_2 - x_1) &= 0 \\ (x_3 - 1) + P(e^{\jmath \pi})(x_2 - x_1) &= 0. \end{aligned}$$

An Example: First Order Controller Design

Available Information:

- Frequency domain data

$$\mathbf{P}(e^{\jmath\omega T}) := \left\{ P(e^{\jmath\omega T}), \omega = \frac{\pi}{T} \text{ sampled every } T = 0.01 \right\}.$$

An Example: First Order Controller Design

Available Information:

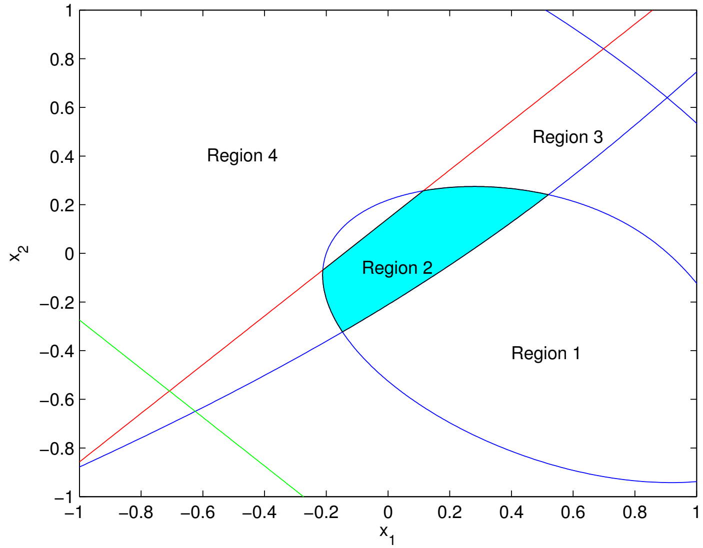
- Frequency domain data

$$\mathbf{P}(e^{\jmath\omega T}) := \left\{ P(e^{\jmath\omega T}), \omega = \frac{\pi}{T} \text{ sampled every } T = 0.01 \right\}.$$

- The plant is stable, i.e., the number of poles outside the unit circle is 0, that is, $o_p = 0$.

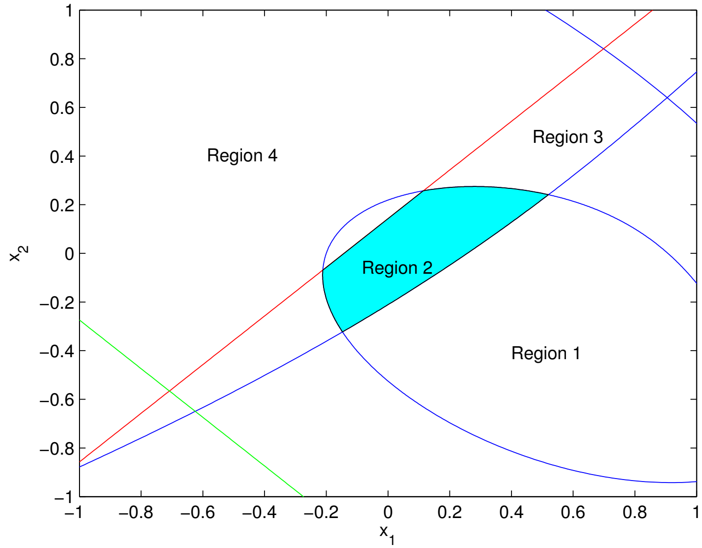
An Example: First Order Controller Design (continue ...)

- At $x = 0.75$, the following region is obtained.



An Example: First Order Controller Design (continue ...)

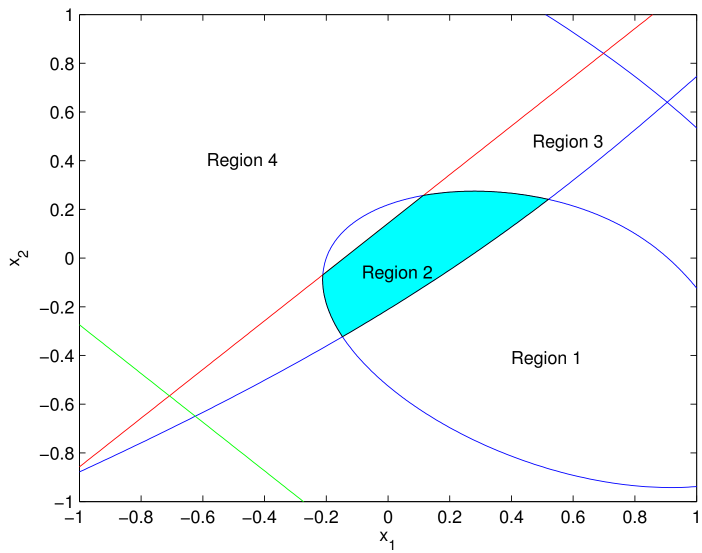
- At $x = 0.75$, the following region is obtained.



- Each separated region represents a set of controller parameters that gives a fixed number of unstable poles of the closed-loop system.

An Example: First Order Controller Design (continue ...)

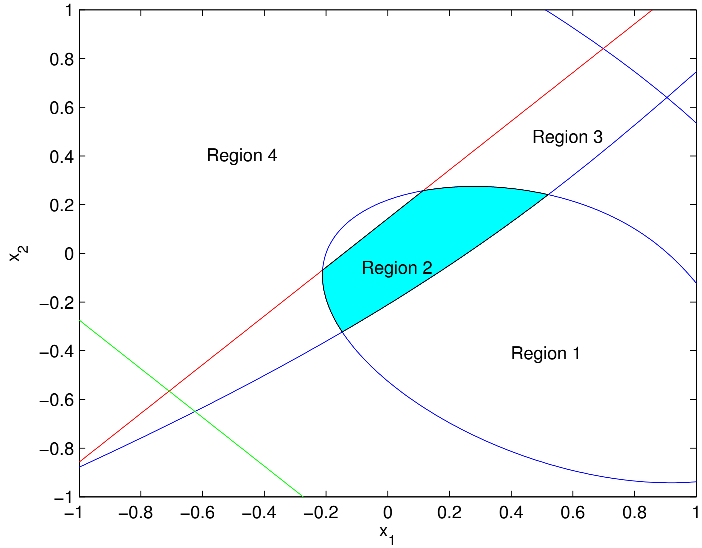
- At $x = 0.75$, the following region is obtained.



- Each separated region represents a set of controller parameters that gives a fixed number of unstable poles of the closed-loop system.
- To identify the stabilizing region, we arbitrarily select a point from each region and plot the corresponding Nyquist plot

An Example: First Order Controller Design (continue ...)

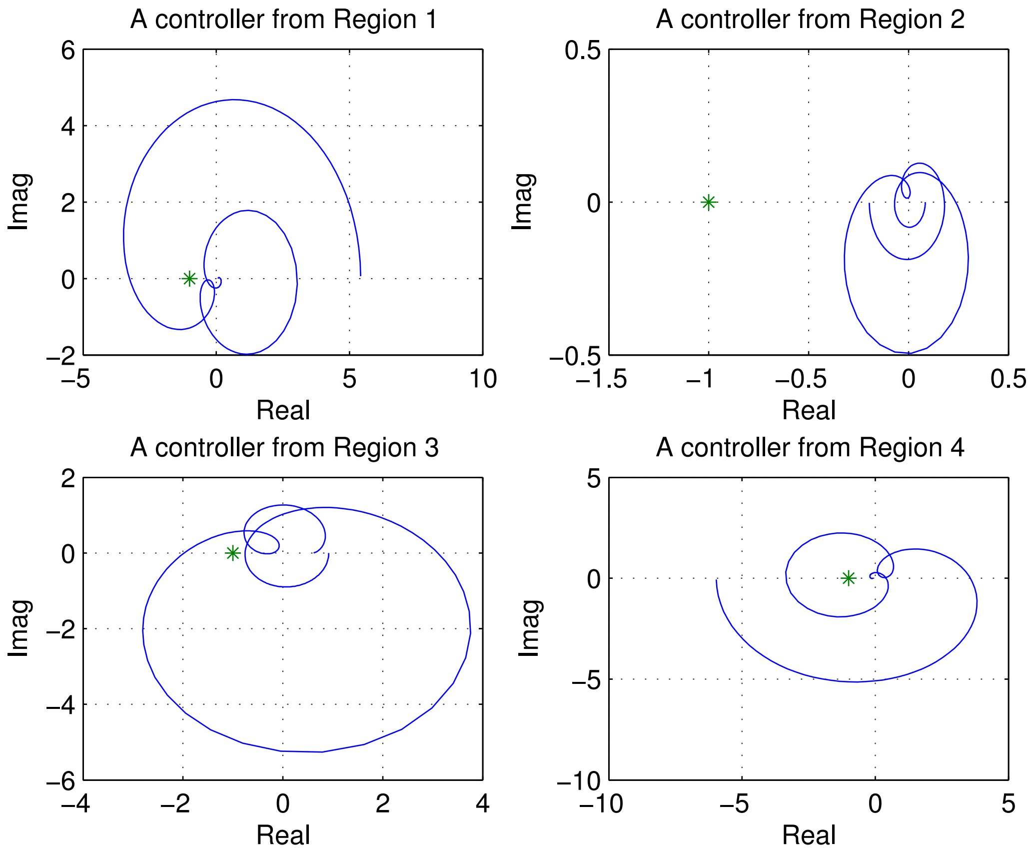
- At $x = 0.75$, the following region is obtained.



- Each separated region represents a set of controller parameters that gives a fixed number of unstable poles of the closed-loop system.
- To identify the stabilizing region, we arbitrarily select a point from each region and plot the corresponding Nyquist plot

An Example: First Order Controller Design (continue ...)

The following figure shows the Nyquist plots with selected controllers from the four specified regions.



An Example: First Order Controller Design (continue ...)

- The Nyquist plot with a controller from Region 1 shows that the encirclement around -1 point is 2π, i.e., $N = 2$.

An Example: First Order Controller Design (continue ...)

- The Nyquist plot with a controller from Region 1 shows that the encirclement around -1 point is 2π, i.e., $N = 2$.

- Since $o_p = 0$, the corresponding closed-loop system will have 2 poles outside the unit circle.

An Example: First Order Controller Design (continue ...)

- The Nyquist plot with a controller from Region 1 shows that the encirclement around -1 point is 2π, i.e., $N = 2$.
- Since $o_p = 0$, the corresponding closed-loop system will have 2 poles outside the unit circle.
- Similarly, corresponding closed-loops system with controllers from Region 3 and 4 will have 2 and 3 poles outside the unit circle, respectively.

An Example: First Order Controller Design (continue ...)

- The Nyquist plot with a controller from Region 1 shows that the encirclement around -1 point is 2π, i.e., $N = 2$.

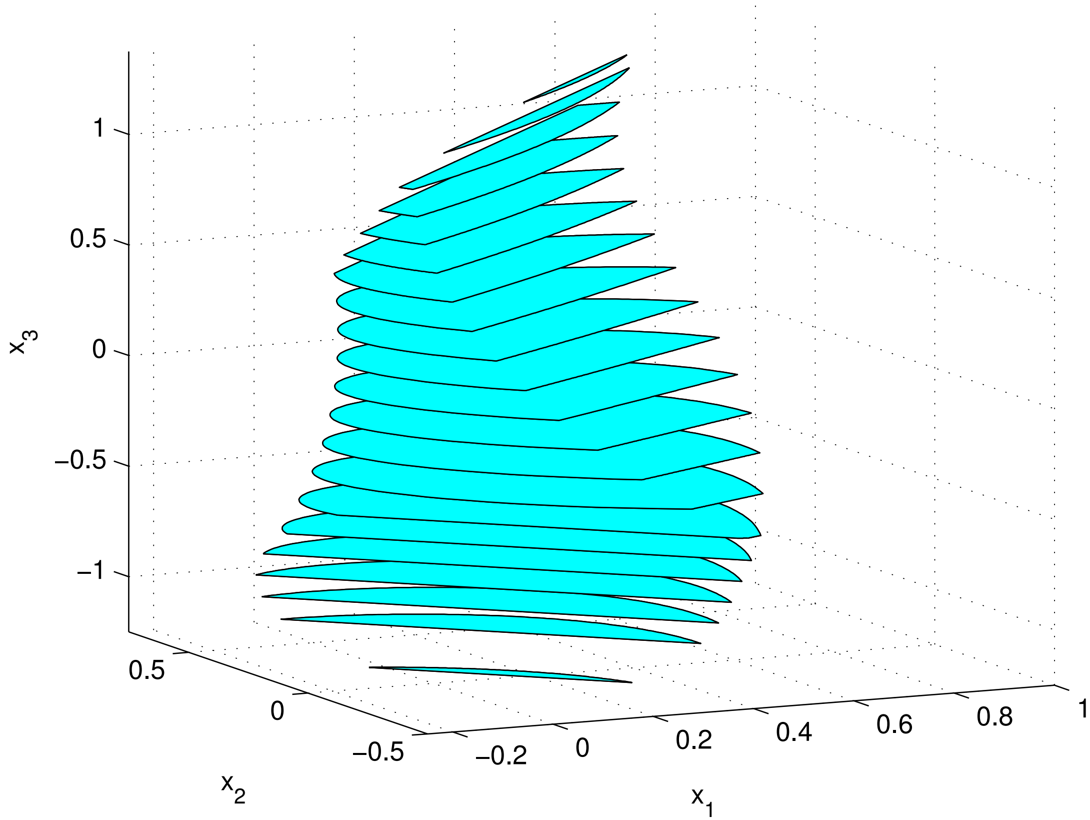
- Since $o_p = 0$, the corresponding closed-loop system will have 2 poles outside the unit circle.

- Similarly, corresponding closed-loops system with controllers from Region 3 and 4 will have 2 and 3 poles outside the unit circle, respectively.

- This test led us to the conclusion that the region 2 is the *only* stabilizing controller parameter region.

An Example: First Order Controller Design (continue ...)

By sweeping over x_3, we have the entire first order stabilizing controllers for the given plant.



Data Measurement by Impulse Response

- Typically data is obtained from the measurement by a sinusoidal input.

Data Measurement by Impulse Response

- Typically data is obtained from the measurement by a sinusoidal input.
- Alternative input - Impulse $\Rightarrow$ Output - Markov parameters

$$y[k] = [m_0, m_1, m_2, \cdots, m_k, \cdots]$$

Data Measurement by Impulse Response

- Typically data is obtained from the measurement by a sinusoidal input.
- Alternative input - Impulse $\Rightarrow$ Output - Markov parameters

$$y[k] = [m_0, m_1, m_2, \cdots, m_k, \cdots]$$

-

$$Y(z) = m_0 + m_1 z^{-1} + \cdots + m_k z^{-k} + \cdots$$

Data Measurement by Impulse Response

- Typically data is obtained from the measurement by a sinusoidal input.
- Alternative input - Impulse $\Rightarrow$ Output - Markov parameters

$$y[k] = [m_0, m_1, m_2, \cdots, m_k, \cdots]$$

-

$$Y(z) = m_0 + m_1 z^{-1} + \cdots + m_k z^{-k} + \cdots$$

- By truncating the terms, we have approximation of the frequency response of the system.

$$P(z)|_{z=-u+jv} = Y_n(z)|_{z=-u+jv}$$

where n is the number of the terms taken from $Y(z)$.

Example: Data Measurement by Impulse Input

- Markov parameter obtained

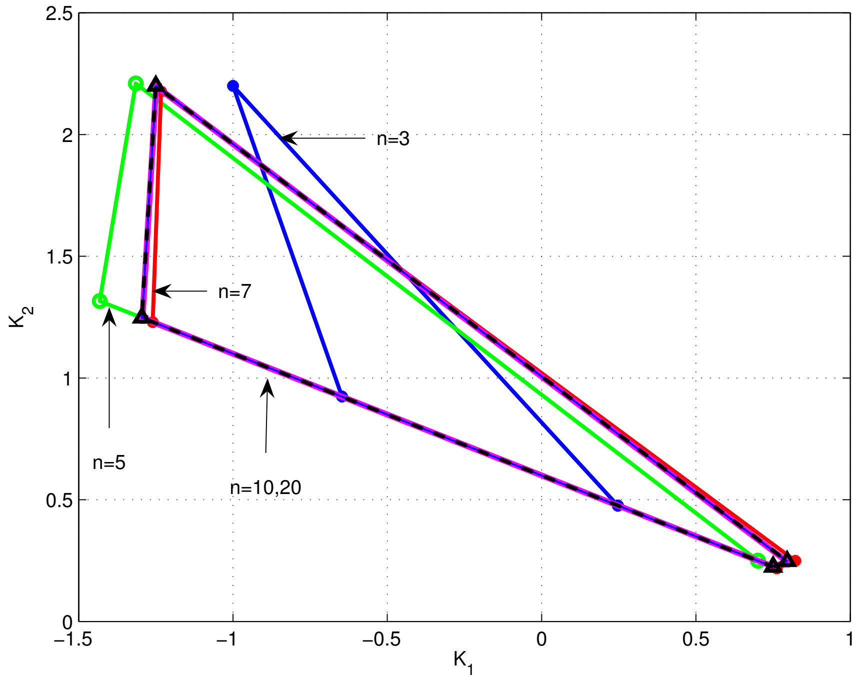
$$y[k] = [0,\ 0,\ 1,\ 0,\ 0.25,\ 0,\ 0.0625,\ 0,\ 0.015625,\ 0,\ 0.00390625,\ \cdots]$$

Example: Data Measurement by Impulse Input

- Markov parameter obtained

$$y[k] = [0,\ 0,\ 1,\ 0,\ 0.25,\ 0,\ 0.0625,\ 0,\ 0.015625,\ 0,\ 0.00390625,\ \cdots]$$

- Stabilizing region with $K_3 = 1.2$ for $n = 3, 5, 7, 10, 20$



Data Measurement by Step Input

- Alternative input - Step Input $\Rightarrow$ Step Response parameters

$$y_s[k] = [y_0, y_1, y_2, \cdots, y_k, \cdots]$$

Data Measurement by Step Input

- Alternative input - Step Input $\Rightarrow$ Step Response parameters

$$y_s[k] = [y_0, y_1, y_2, \cdots, y_k, \cdots]$$

-

$$\begin{aligned} H(z)|_{z=-u+jv} &= Y_s(z)\left[\frac{z-1}{z}\right]\Bigg|_{z=-u+jv} \\ &= \left(y_0 + y_1 z^{-1} + y_2 z^{-2} + \cdots + y_k z^{-k} + \cdots\right)\left(1 - z^{-1}\right)|_{z=-u+jv} \\ &= y_0 + (y_1 - y_0)z^{-1} + (y_2 - y_1)z^{-2} + \cdots + (y_k - y_{k-1})z^{-k} + \cdots \\ &= m_0 + m_1 z^{-1} + m_2 z^{-2} + \cdots + m_k z^{-k} + \cdots \end{aligned}$$

where $m_0, m_1, \cdots, m_k, \cdots$ are the Markov parameters.

Data Measurement by Step Input

- Alternative input - Step Input $\Rightarrow$ Step Response parameters

$$y_s[k] = [y_0, y_1, y_2, \cdots, y_k, \cdots]$$

-

$$H(z)|_{z=-u+\jmath v} = Y_s(z)\left[\frac{z-1}{z}\right]\Big|_{z=-u+\jmath v}$$

$$= \left(y_0 + y_1 z^{-1} + y_2 z^{-2} + \cdots + y_k z^{-k} + \cdots\right)(1 - z^{-1})\,|_{z=-u+\jmath v}$$

$$= y_0 + (y_1 - y_0)z^{-1} + (y_2 - y_1)z^{-2} + \cdots + (y_k - y_{k-1})z^{-k} + \cdots$$

$$= m_0 + m_1 z^{-1} + m_2 z^{-2} + \cdots + m_k z^{-k} + \cdots$$

where $m_0, m_1, \cdots, m_k, \cdots$ are the Markov parameters.

- By truncating the terms, we have approximation of the frequency response of the system.

$$P(z)|_{z=-u+\jmath v} = H_n(z)|_{z=-u+\jmath v}$$

where n is the number of the terms taken from $H(z)$.

Example: Data Measurement by Step Input

- The step response and the Markov parameters computed

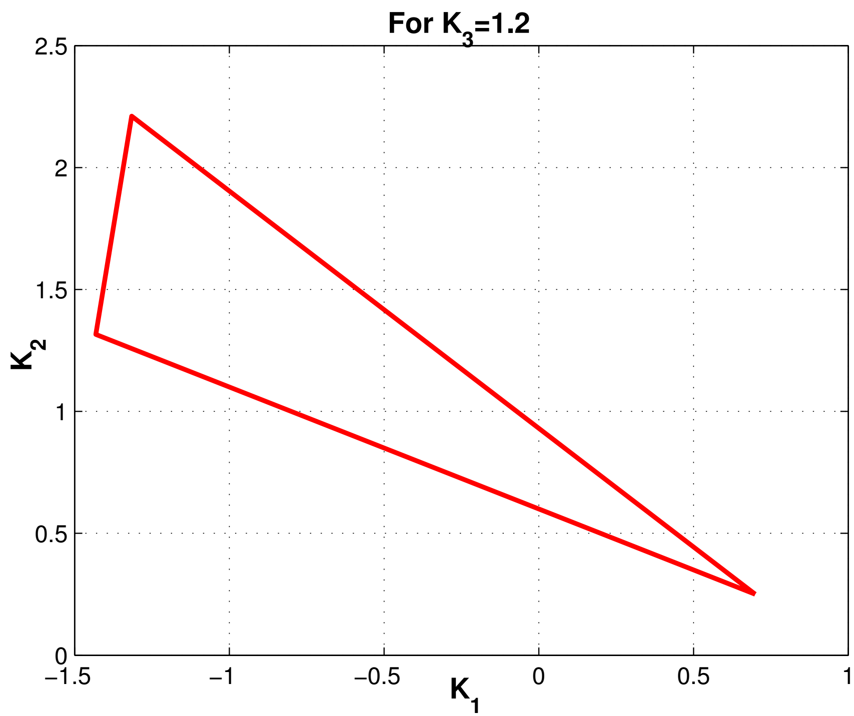
$$
\begin{aligned}
y_s[k] &= [0,\ 0,\ 1,\ 1,\ 1.25,\ 1.25,\ 1.3125,\ 1.3125,\ 1.328125,\ 1.328125,\ 1.33203125,\ \cdots] \\
m(k) &= [0,\ 0,\ 1,\ 0,\ 0.25,\ 0,\ 0.0625,\ 0.015625,\ 0,\ 0.00390625,\cdots]
\end{aligned}
$$

Example: Data Measurement by Step Input

- The step response and the Markov parameters computed

$$
\begin{aligned}
y_s[k] &= [0,\ 0,\ 1,\ 1,\ 1.25,\ 1.25,\ 1.3125,\ 1.3125,\ 1.328125,\ 1.328125,\ 1.33203125,\ \cdot \\
m(k) &= [0,\ 0,\ 1,\ 0,\ 0.25,\ 0,\ 0.0625,\ 0.015625,\ 0,\ 0.00390625, \cdots]
\end{aligned}
$$

- Stabilizing resion with $K_3 = 1.2$ for $n = 5$



Computer Aided Design using Labview

- The algorithm for the design of a PID Controller from the frequency response data of the system has programmed in LabVIEW due to its user-friendly graphical environment.

Computer Aided Design using Labview

- The algorithm for the design of a PID Controller from the frequency response data of the system has programmed in LabVIEW due to its user-friendly graphical environment.
- The Virtual Instrument (VI) in Labview have a front panel that is displayed to the user and a block diagram, where the computations are performed.

Computer Aided Design using Labview

- The algorithm for the design of a PID Controller from the frequency response data of the system has programmed in LabVIEW due to its user-friendly graphical environment.
- The Virtual Instrument (VI) in Labview have a front panel that is displayed to the user and a block diagram, where the computations are performed.
- The inputs to the program are the frequency response data and the number of RHP poles of the system.

Computer Aided Design using Labview

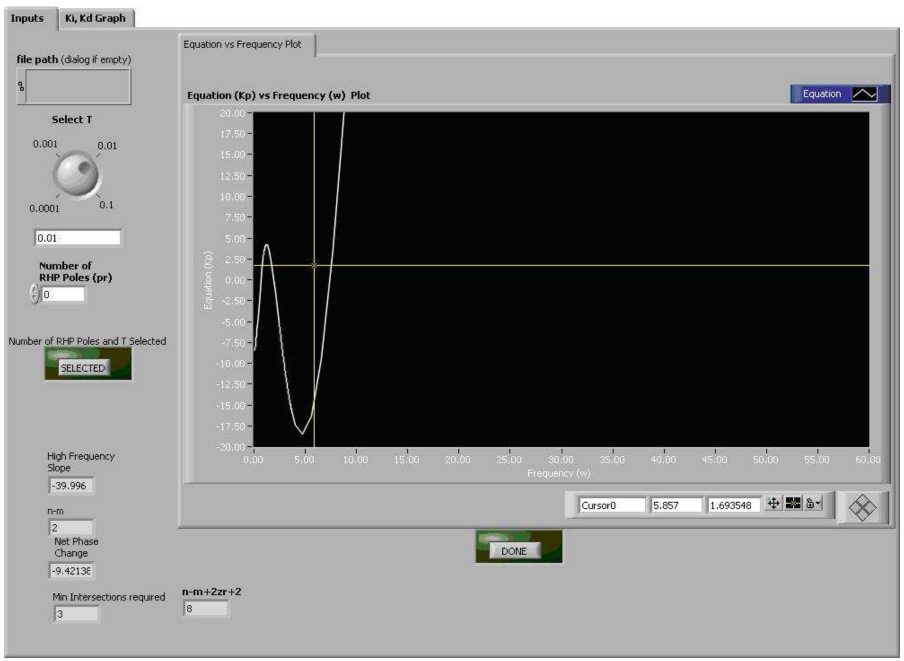
- The algorithm for the design of a PID Controller from the frequency response data of the system has programmed in LabVIEW due to its user-friendly graphical environment.
- The Virtual Instrument (VI) in Labview have a front panel that is displayed to the user and a block diagram, where the computations are performed.
- The inputs to the program are the frequency response data and the number of RHP poles of the system.
- Given these inputs, the entire range of K_p that can stabilize the system is displayed,

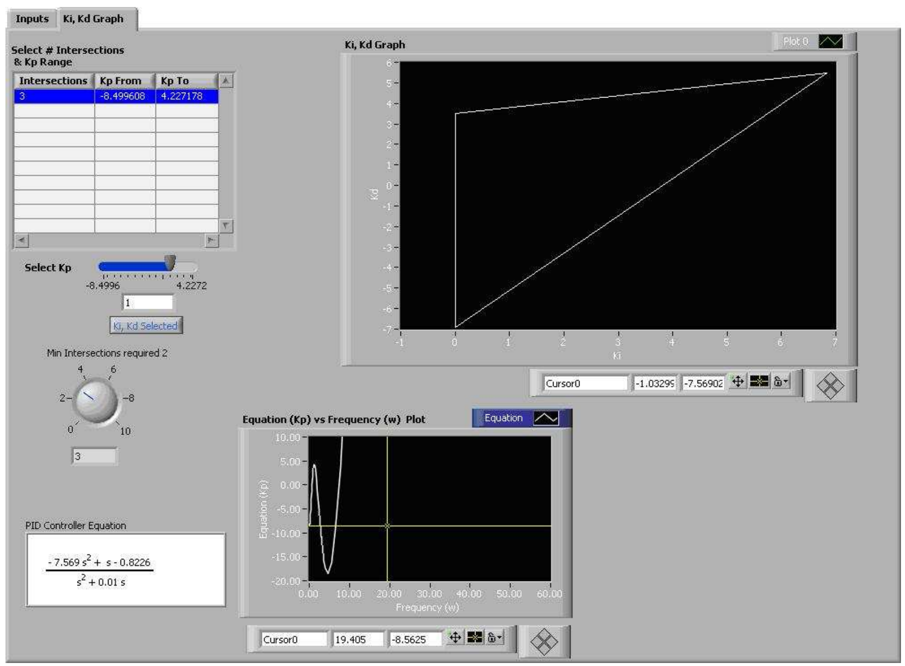
Computer Aided Design using Labview

- The algorithm for the design of a PID Controller from the frequency response data of the system has programmed in LabVIEW due to its user-friendly graphical environment.
- The Virtual Instrument (VI) in Labview have a front panel that is displayed to the user and a block diagram, where the computations are performed.
- The inputs to the program are the frequency response data and the number of RHP poles of the system.
- Given these inputs, the entire range of K_p that can stabilize the system is displayed,
- and as the user scrolls through the stabilizing range of K_p, entire stabilizing ranges of K_i and K_d are displayed.

CAD using LABVIEW (continue...)

- Left: The file containing the frequency response data from a stable system (number of right hand plane poles equals zero) is fed into the program through the file path box located at the top.
- Right: The front panel shows stabilizing sets of K_p, K_i and K_d.





CAD using LABVIEW (continue...)

- Left: T is set to a very small number.

CAD using LABVIEW (continue...)

- Left: T is set to a very small number.
  - $g(\omega)$ versus frequency generated from the inputs is used to compute ω values associated with a particular value of K_p.

CAD using LABVIEW (continue...)

- Left: T is set to a very small number.
  - $g(\omega)$ versus frequency generated from the inputs is used to compute ω values associated with a particular value of K_p.
  - Using the values of ω obtained above, linear equations are solved by the program to compute stabilizing sets of K_i and K_d for a fixed K_p.

CAD using LABVIEW (continue...)

- Left: T is set to a very small number.
  - $g(\omega)$ versus frequency generated from the inputs is used to compute ω values associated with a particular value of K_p.
  - Using the values of ω obtained above, linear equations are solved by the program to compute stabilizing sets of K_i and K_d for a fixed K_p.
  - As the selected K_p is changed, the K_i, K_d graph changes dynamically to show the new stabilizing ranges of K_i and K_d for the selected K_p.

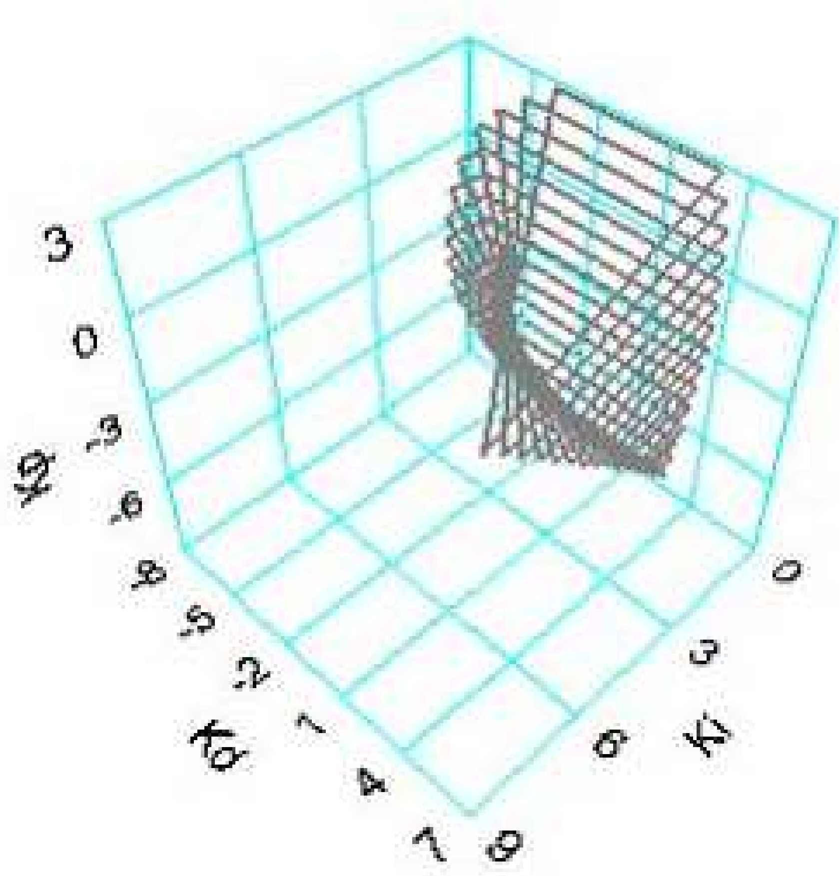
CAD using LABVIEW (continue...)

- Left: T is set to a very small number.
  - $g(\omega)$ versus frequency generated from the inputs is used to compute ω values associated with a particular value of K_p.
  - Using the values of ω obtained above, linear equations are solved by the program to compute stabilizing sets of K_i and K_d for a fixed K_p.
  - As the selected K_p is changed, the K_i, K_d graph changes dynamically to show the new stabilizing ranges of K_i and K_d for the selected K_p.

- Right: Gives the stabilizing range of K_i and K_d for $K_p = 1$.

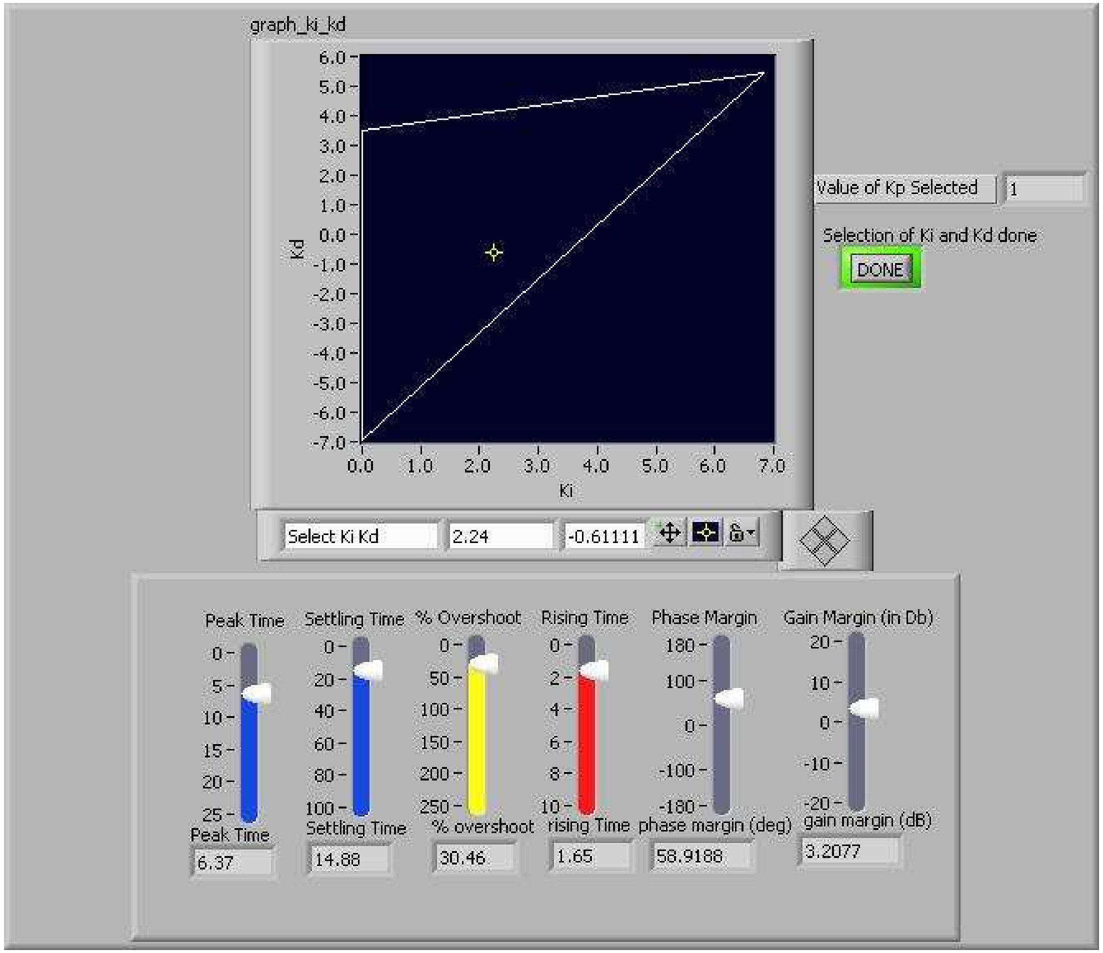
CAD using LABVIEW: 3D graph of stabilizing sets

The stabilizing set of K_p, K_i and K_d for the given system is shown.



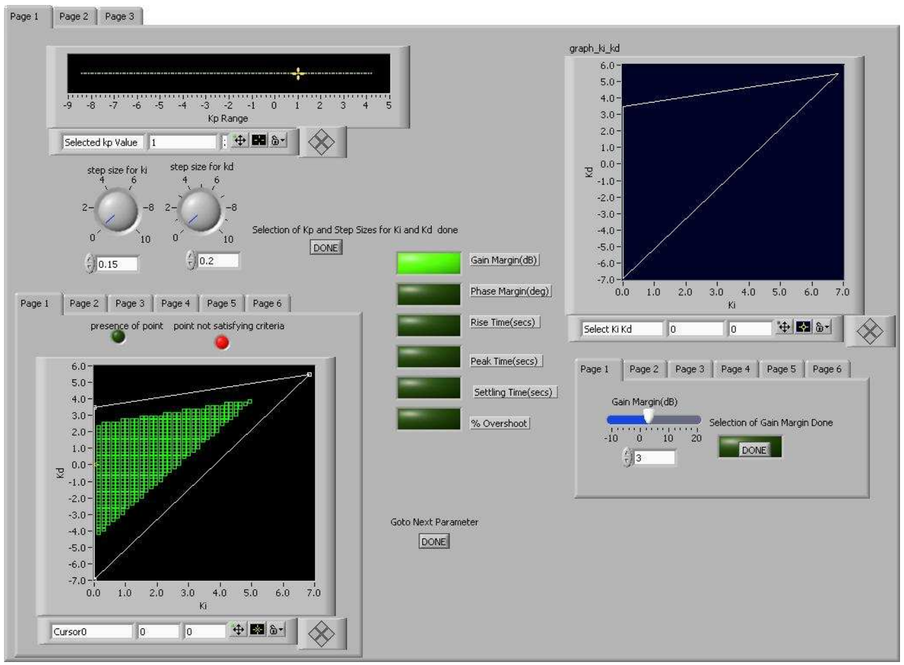
CAD using LABVIEW (continue...)

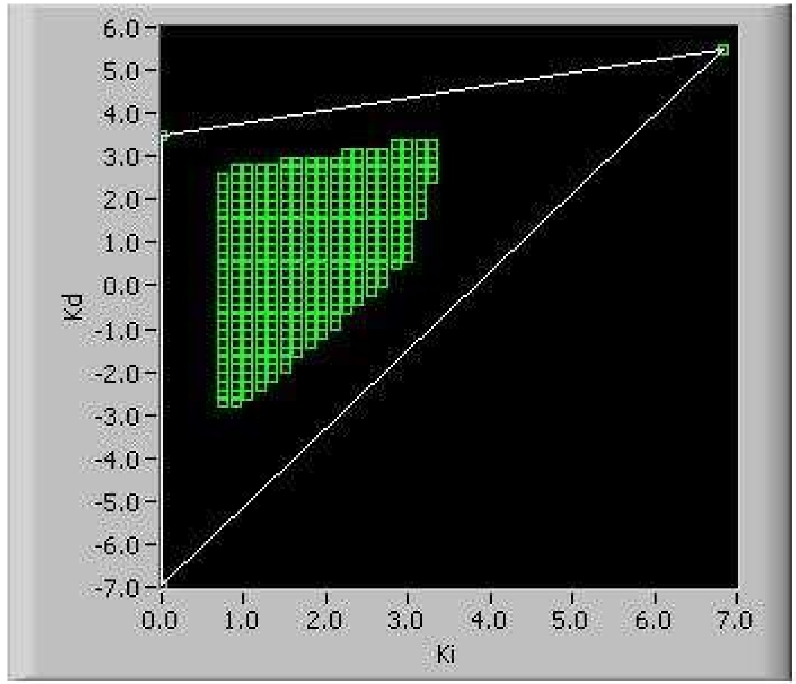
Front Panel of VI showing performance indices for specific K_p, K_i and K_d.



CAD using LABVIEW (continue...)

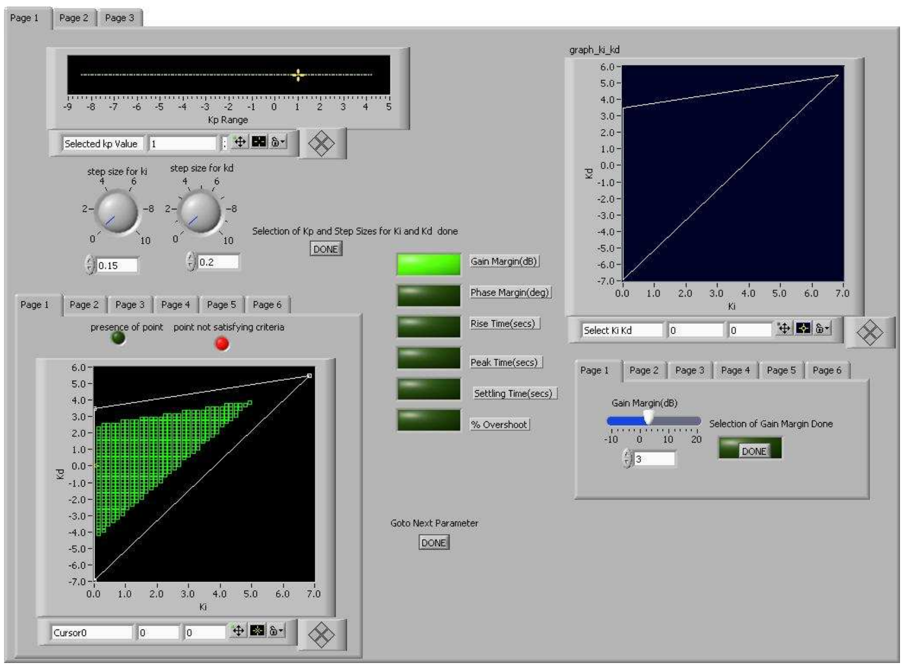
- Front Panel of VI that satisfies multiple performance index specifications

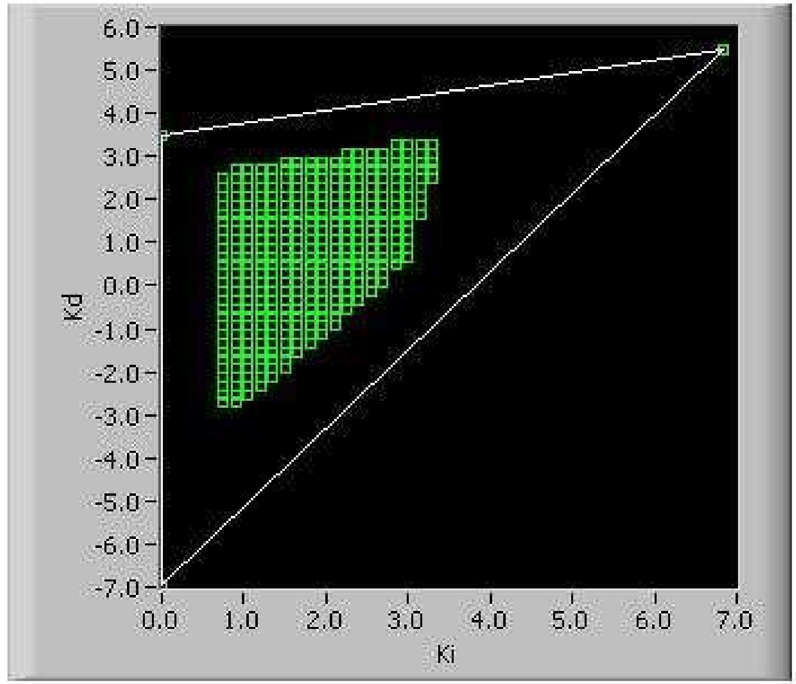




CAD using LABVIEW (continue...)

- Front Panel of VI that satisfies multiple performance index specifications





- The generated set of points satisfies multiple performance indices simultaneously. Performance indices used in the program: Gain Margin$= 3$db, Phase Margin$= 45^o$, Overshoot$= 30\%$.

Concluding Remarks

- We have shown that the complete set of PID/ First Order stabilizing controllers achieving stability and various meaningful performance specifications can be found from the frequency response of the plant and knowledge of the number of RHP plant poles.

Concluding Remarks

- We have shown that the complete set of PID/ First Order stabilizing controllers achieving stability and various meaningful performance specifications can be found from the frequency response of the plant and knowledge of the number of RHP plant poles.

- Note that

Concluding Remarks

- We have shown that the complete set of PID/ First Order stabilizing controllers achieving stability and various meaningful performance specifications can be found from the frequency response of the plant and knowledge of the number of RHP plant poles.

- Note that
  - that this calculation can be done by a nested linear programming procedure and

Concluding Remarks

- We have shown that the complete set of PID/ First Order stabilizing controllers achieving stability and various meaningful performance specifications can be found from the frequency response of the plant and knowledge of the number of RHP plant poles.

- Note that
  - that this calculation can be done by a nested linear programming procedure and
  - that only knowledge of the frequency response and number of RHP poles is sufficient.

Concluding Remarks (continue...)

- It is not clear whether these advantages can be extended to other types of controllers and this is an area worth investigating since determining such stabilizing and performance sets is an important step toward lower order, robust and high performance controller design

Concluding Remarks (continue...)

- It is not clear whether these advantages can be extended to other types of controllers and this is an area worth investigating since determining such stabilizing and performance sets is an important step toward lower order, robust and high performance controller design

- It is also worth investigating how this procedure can be modified to accommodate incomplete or finite frequency data.

Concluding Remarks (continue...)

- It is not clear whether these advantages can be extended to other types of controllers and this is an area worth investigating since determining such stabilizing and performance sets is an important step toward lower order, robust and high performance controller design
- It is also worth investigating how this procedure can be modified to accommodate incomplete or finite frequency data.
- An important area of research is MIMO PID control and the extension of the results given here to the multi-variable case.

End of Presentation

Thank You

The short course lectures are based on materials contained in the following references:

- A. Datta, M.T. Ho, and S.P. Bhattacharyya, Structure and Synthesis of PID Controllers, Advances in Industrial Control, Springer, 2000.
- G.J. Silva, A. Datta, and S.P. Bhattacharyya, PID Controllers for Time-Delay Systems, Birkhäuser, 2004.
- S.P. Bhattacharyya, A. Datta, and L.H. Keel, Linear Control Theorey: Structure, Robustness, and Optimization, CRC Press, 2009.
- L.H. Keel, Y.C. Kim, and S.P. Bhattacharyya, Tutorial Workshop on Advances in Three Term Control, 17th IFAC World Congress, Seoul, Korea, July 6 – 11, 2008.

- L.H. Keel, J.I. Rego, and S.P. Bhattacharyya, "A new approach to digital PID controller design," IEEE Transactions on Automatic Control, Vol. 48, No. 4, pp. 687 - 692, April 2003.
- R.N. Tantaris, L.H. Keel, and S.P. Bhattacharyya, "Stabilization of discrete-time systems by first order controllers," IEEE Transactions on Automatic Control, Vol. 48, No. 5, pp. 858 - 861, May 2003.
- R.N. Tantaris, L.H. Keel, and S.P. Bhattacharyya, "H1 design with first order controllers," IEEE Transactions on Automatic Control, Vol. 51, No. 8, pp. 1343 - 1347, August, 2006.
- L.H. Keel, S. Mitra, and S.P. Bhattacharyya, "Data driven synthesis of three term digital controllers," SICE Journal of Control, Measurement, and System Integration, Vol. 1, No. 2, pp. 102 - 110, March 2008.
- L.H. Keel and S.P. Bhattacharyya, "Controller synthesis free of analytical models: three term controllers," IEEE Transactions on Automatic Control, Vol. 53, No. 6, pp. 1353 - 1369, July 2008.