A Hybrid Control Scheme for a Copier Paperpath [1]

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Abstract

This paper presents a hybrid control approach for a copier paperpath. The paperpath transports sheets from the feeder to the image transfer section. At arrival time, every sheet must synchronize with its image on the photoreceptor belt. A non-cooperative hybrid control scheme that controls intersheet spacings is proposed. The controller switches between different modes to satisfy all the system constraints. Cooperation, which enhances performance, is then introduced for a simplified system model, based on the notion of controllable regions. Simulations illustrate both strategies.

1. System Overview

The system we consider is the paperpath of a cut sheet copier [1], shown in figure 1. Several independently driven sections transport sheets from the feeder to the image transfer section, where the image (toner particles stuck to the photo-receptor belt) is transferred onto the sheet.

The overall goal of the control system is to make sure that each sheet is matched with the position and velocity of its image on the photo-receptor belt, which runs at constant speed, v_d. If a position or velocity error exists at the desired image transfer time, the image will be smeared or not positioned correctly onto the sheet.

The system dynamics consists of two parts: **1)** Section dynamics: map system inputs to section velocities, **2)** Paperpath dynamics: a system of integrators with hybrid characteristics [2] which map section velocities to sheet positions.

If the section dynamics are ignored, the system inputs are the section velocities. This model is referred to below as the $1/s$-model. On the other hand, since the sections are driven by current-controlled DC-motors, a more realistic model of the section dynamics is that of a single integrator. In this case, the overall system inputs are the motor currents, which approximate the section accelerations q. This model is referred to below as the $1/s^2$-model.

2. Why intersheet spacing control?

This paper forms an extension of the intersheet spacing control (ISSC) approach in [1]. ISSC tries to maintain a desired spacing, d_s, between neighoring sheets. d_s corresponds to the spacing between images on the photoreceptor belt. An alternative approach is absolute reference tracking control (ARTC), where each sheet tracks an individual reference trajectory [3] .

ARTC requires reference trajectories that satisfy the system constraints. In case a disturbance enters the system, all trajectories upstream may have to be updated. This is automatically satisfied in ISSC.

Note also that a section can contain multiple sheets, which can further complicate the problem. ISSC differs from ARTC when correcting the intersheet spacing error between the most downstream sheet in a section and its downstream neighbor in the next section. For ISSC, this action does *not* introduce errors for the upstream sheets in the section. For ARTC however, that correction will cause the upstream sheets in that section to deviate from their reference trajectories.

Experiments at Xerox Corporation have shown that the largest errors occur when a sheet enters the paperpath, either from the feeder or a duplex unit. Disturbances along the paperpath are relatively small.

With this in mind, an actuator hierarchy is proposed such that actuator capabilities (in terms of maximum/minimum velocity and acceleration) are largest at the beginning of the paperpath and decrease towards the end. This way, an upstream section can always reduce a spacing error *regardless* of the control action of the downstream section. So initial errors will be reduced gradually as the sheet travels through the paperpath.

The image transfer section cannot be controlled. Furthermore, the spacing error in the last section is most important since it determines the final position/velocity error when the sheet enters the image transfer section. Therefore, we choose to feedforward control actions in the upstream direction. Note that by doing so, the correction of an error downstream does not introduce an error upstream. Also, when two sections synchronize during sheet transfer, the downstream section dictates the velocity since the last section will have to synchronize with the uncontrollable image transfer section. This section hierarchy goes in the opposite direction of the actuator hierarchy.

3. Dynamic funnel tracking control

While a sheet is being transferred between two sections, both sections must run at equal velocity to avoid buckling or tearing of the sheet. In accordance with the section hierarchy, the velocity during synchronization is that of the downstream section.

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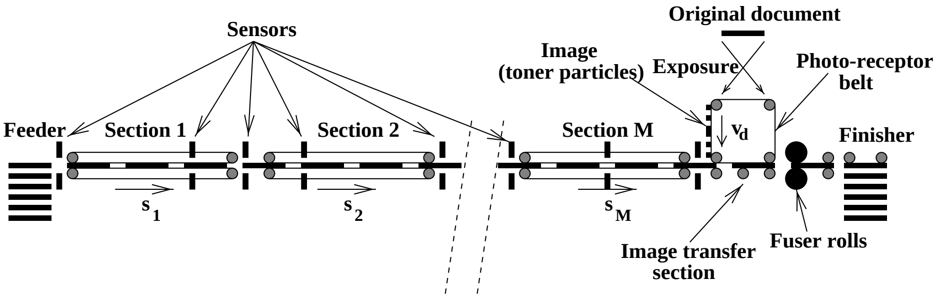


Figure 1: Basic overview of the paperpath and other copier subsystems

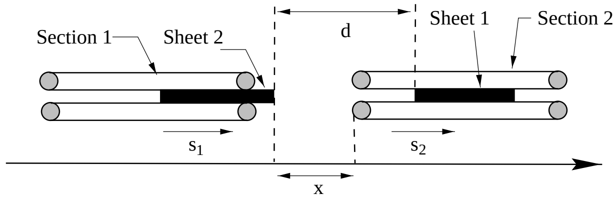


Figure 2: Initial Conditions and Nomenclature

Consider the situation illustrated in figure 2. The discussion below assumes section 1 has *no knowledge* about the control action for section 2. Sheet 2 is at a distance x from the entrance of section 2. Assume the spacing between both sheets is too large. Section 1 must decrease the error by running faster than section 2. However, when sheet 2 enters section 2, both velocities must be equal again. The problem at hand is to determine an allowable velocity range for section 1, as a function of x and the velocity of section 2, such that *synchronization can be guaranteed* when $x = 0$.

3.1. The velocity funnel

As discussed in section 2, the section velocities s and accelerations q satisfy

$$\begin{array}{ll} s_{1,max} > s_{2,max} > 0 & 0 < s_{1,min} < s_{2,min} \\ q_{1,max} > q_{2,max} > 0 & q_{1,min} < q_{2,min} < 0 \end{array} \tag{1}$$

Assume the worst case scenario for the initial conditions mentioned above ($s_1 > s_2$), i.e. section 2 starts to slow down with maximum deceleration, $q_{2,min}$. The *maximum* allowable velocity for section 1 at that point, $\bar{s}_1(x, s_2)$, is such that if section 1 also decelerates at maximum pace, s_1 equals s_2 when $x = 0$. Depending on the initial conditions, s_2 may saturate at $s_{2,min}$ before $x = 0$. The two possible scenarios are illustrated in figure 3. After some calculations, one obtains

$$\bar{s}_1 = \frac{\alpha_1\beta_1 s_2 + \sqrt{(\alpha_1\beta_1 s_2)^2 + (1-\beta_1^2)(\alpha_1^2 s_2^2 - 2q_{1,min}x)}}{1-\beta_1^2} \tag{2}$$

$$\alpha_1 = \frac{q_{1,min}}{q_{1,min} - q_{2,min}} \qquad \beta_1 = \frac{-q_{2,min}}{q_{1,min} - q_{2,min}} = 1 - \alpha_1 \tag{3}$$

if section 2 does not saturate before $x = 0$ and

$$\bar{s}_1(x) = \sqrt{s_{2,min}^2 - 2q_{1,min}x} \tag{4}$$

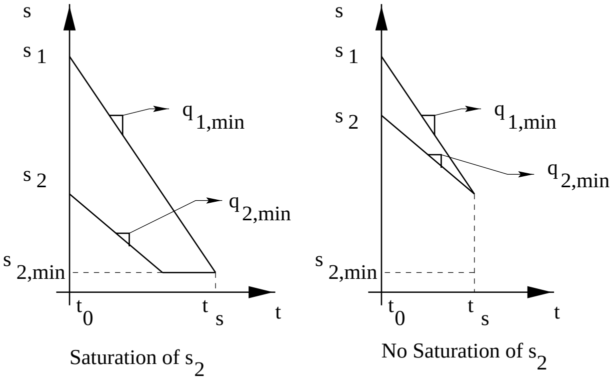


Figure 3: Worst case assumption for $s_1 > s_2$

otherwise. Saturation will occur if initially

$$s_2 + q_{2,min}\left(\frac{s_1 - s_2}{q_{2,min} - q_{1,min}}\right) < s_{2,min} \tag{5}$$

Analogous expressions exist for the lower bound on s_1.

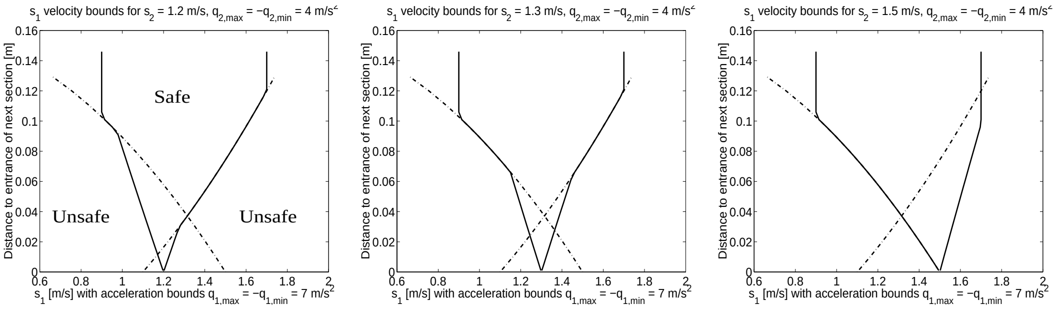
Some typical velocity boundaries for different constant values of s_2 are shown in figure 4. The horizontal axis represents s_1, the vertical axis x. The allowable velocity range for section 1 decreases as its leading sheet approaches section 2. When the sheet finally enters, both sections synchronize.

The velocity limits can be compared to a *funnel.* The dashed lines correspond to the saturation case. The part of the tip of the funnel not on the dashed lines corresponds to the no saturation case. Notice how the shape of the funnel changes as the velocity of the downstream section changes. Hence the nomenclature *dynamic funnel.*

3.2. Spacing control with funnel constraint

The funnel limits the allowable velocity range as a sheet approaches the next section. If the sheet velocity remains *inside* the funnel, the sections will synchronize at sheet transfer, *regardless* of the control action for the downstream section.

The main control goal in intersheet spacing control is to obtain a desired spacing between sheets. Clearly, if a sheet is running late, a section will speed up to decrease the spacing between the sheet and its downstream neighbor in the next section. However, as discussed above, a section cannot run arbitrarily fast in order to always guar-



$s_2 = 1.2$ m/s $\quad$ $s_2 = 1.3$ m/s $\quad$ $s_2 = 1.5$ m/s

Figure 4: Funnel examples for constant s_2

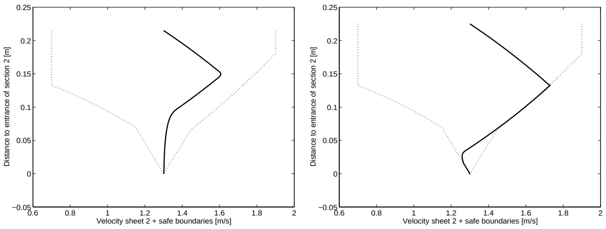


Figure 5: s_1 trajectory before sheet transfer

antee synchronization. This is a conflict of interest. The position controller requests a speed increase to decrease the spacing error, but by doing so, possibly violates the funnel constraint.

The funnel constraint is more important than the spacing error. Indeed, the remainder of an error can always be corrected in a downstream section, but synchronization cannot be postponed. If the funnel constraint is violated for the last section in the machine, the sheet will not arrive with zero error (note that the last section has a slightly different funnel since the image transfer section runs at constant velocity).

The proposed control strategy consists of switching between two different control modes. Position control is active as long as the position controller keeps the section velocity within the funnel. If the velocity exits the funnel, the controller switches to *dynamic funnel tracking control*. In this mode, the section velocity tracks the boundary of the funnel until the position control input points again towards the inside of the funnel. Indeed, it may not be needed to remain on the funnel boundary if the spacing error has become sufficiently small. Note that to enable tracking from outside the funnel, the funnel can be designed assuming only 95% of the available acceleration capabilities.

This strategy reduces the spacing error *as much as possible* while ensuring section synchronization at arrival time. Figure 5 shows two examples. The solid line shows the evolution of s_1 for an initial positive spacing error. s_2 is constant. The case on the left has a smaller initial error. Section 1 corrects the error before $x = 0$ so the sections synchronize early. For the larger spacing error, s_1 hits the funnel twice. Both sections synchronize at $x = 0$.

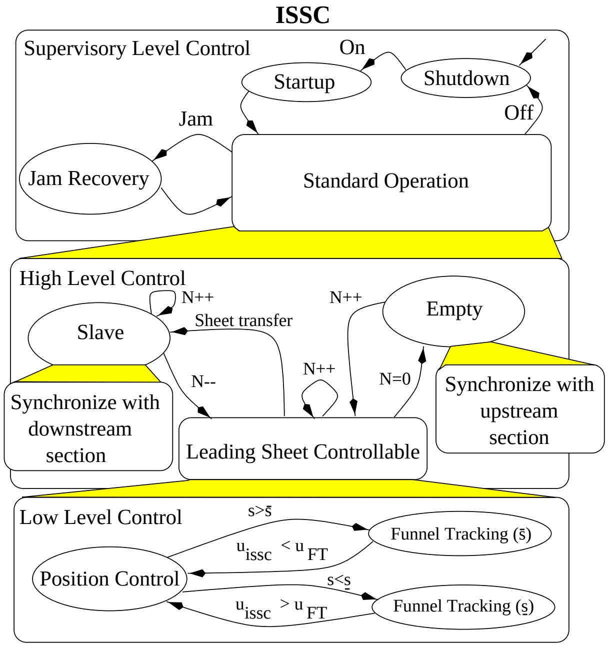


Figure 6: Non cooperative ISSC controller flowchart

4. A hybrid control scheme for ISSC

This section presents a hierarchical hybrid control scheme for the copier paperpath, based on the $1/s^2$-model (see section 1). The strategy switches between different control modes in order to satisfy the system constraints and is illustrated in figure 6. It is not cooperative. Each section only considers the spacing error of its leading sheet, regardless of other errors in the paperpath.

4.1. Supervisory Level Control

The supervisory level handles the state of the section. Error checking is performed at startup. When the section receives a shutdown signal, it decelerates to a stop. The jam recovery mode, which will form the subject of future research, requires a totally different control scheme and is therefore implemented as a separate mode. Finally, the default mode is standard operation,which is further detailed below.

4.2. High Level Control

The default mode consists of three high level control actions. If no sheet is present inside the section, it tracks the velocity of the upstream section, possibly subject to saturation. This makes it easier for the upstream section to satisfy the synchronization constraint while doing position control.

In the same way, when the section is transferring a sheet to a downstream section, it becomes the slave of that section in accordance with the section hierarchy structure. Therefore, it must synchronize with that section.

In all other cases, the leading sheet in the section can be controlled. The actual control action is described in the low level control block. Note that a counter N is used to keep track of the number of sheets inside every section.

4.3. Low Level Control

The default low level control action is position control. As mentioned in section 2, position control consists of a feedforward signal from the downstream section and a feedback loop to correct the remaining position error. A simple choice for this feedback loop for a double integrator model is Proportional-Derivative control.

The position controller, however, does not take the synchronization constraints into account. Therefore, funnel shaped limitations must be imposed on the allowable section velocity as discussed in section 3. If the section velocity crosses the funnel boundary, the controller switches to dynamic funnel tracking. Position control resumes once the position control action points again towards the inside of the funnel. In this way, section synchronization is always guaranteed while maximizing error reduction.

4.4. ISSC simulation results

Figure 7 illustrates ISSC by showing the evolution of the intersheet spacings d versus sheet positions as sheets 1–4 travel along the paperpath. The grey areas represent sections. In accordance with the actuator hierarchy, the spacing error decreases when a sheet becomes the leading sheet in its section. For the upstream sheets in that section, the intersheet spacing remains constant.

All sheets are fed into the paperpath with some initial spacing error. The errors for sheets 1 and 4 are corrected in section 1. Sheet 2 is at the correct intersheet spacing when leaving section 2. The large initial error of sheet 3 is not cancelled until it leaves section 3.

5. Introducing cooperation among sections

Standard ISSC does not include cooperation. Sections only consider the spacing error of their leading sheet and do not look backwards. The strategy is favorable due to its simplicity, but it is not optimal. Cooperation among sections can remove larger spacing errors. Cooperative intersheet spacing control (COOP ISSC) therefore uses the available resources more effectively.

COOP ISSC can be illustrated with the initial conditions shown in figure 8. Sheet 2 is on time, sheet 3 is too late.

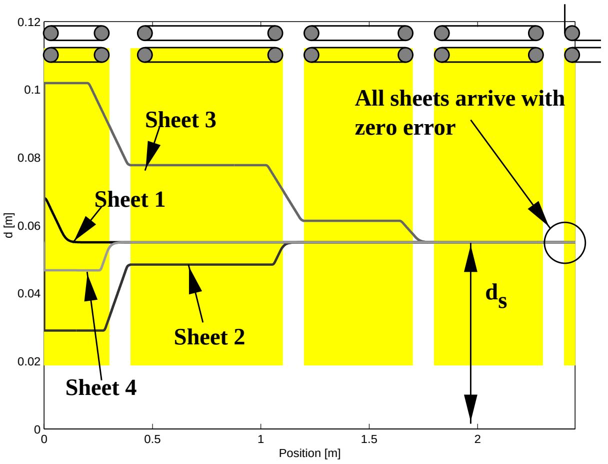


Figure 7: Non-cooperative ISSC

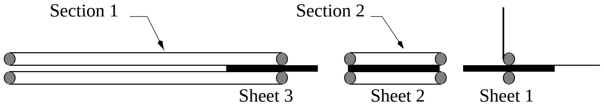


Figure 8: Cooperative ISSC example: initial conditions

Standard ISSC would continue to run sheet 2 at its nominal velocity, v_d, assuming no disturbances. At the same time, section 1 would run at maximum velocity to decrease the spacing error as much as possible before synchronization.

This control strategy can be improved as follows. By only looking downstream, section 2 is not making full use of its actuation capabilities. Sheet 2 has zero spacing error. By continuing to run at v_d, it will make it in time to the image transfer section. However, it could also slow down and speed up again and still arrive in time. By doing so, section 2 helps section 1 to decrease its spacing error.

5.1. Cooperation Decision Logic

The concept of controllable regions is used to implement a safe COOP ISSC strategy. For simplicity reasons, we use the $1/s$-model. The controllable regions for this model, illustrated in figure 9, were derived in [2]. Controllable regions put a *bound on the allowable intersheet spacing* d: $d_{min} < d < d_{max}$. As long as a sheet remains inside the

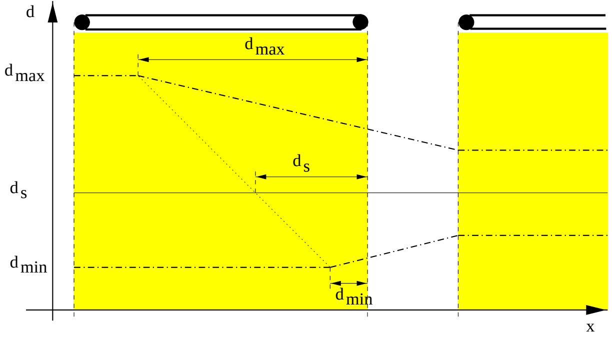


Figure 9: A close-up of the controllable region of a section

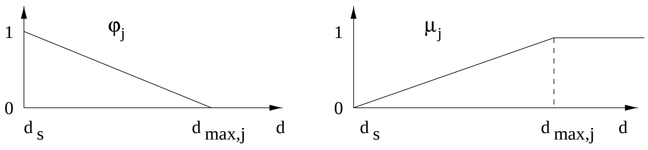


Figure 10: One possible way to assign values to φ_j and μ_j

controllable region, it is physically possible to reduce its position error to zero before it reaches the image transfer section, independent of any downstream control actions. Therefore, one way to *guarantee performance* is to keep every sheet within the controllable region. If it is outside the controllable region, the controller should try to bring the sheet back inside. This reasoning forms the basis for the COOP ISSC strategy.

Define two variables φ_j and μ_j, where

$$\begin{array}{ll} 0 < \varphi_j < 1: & \text{willingness of section j to cooperate,} \\ & \text{i.e. assist section } j-1 \\ 0 < \mu_j < 1: & \text{need of section j for cooperation} \\ & \text{from section } j+1 \end{array} \tag{6}$$

One way to assign values to φ_j and μ_j is illustrated in figure 10. If the leading sheet of a section is on time, i.e. it is at the desired spacing d_s from its downstream neighbor, the section is willing to cooperate, $\varphi_j = 1$ and does not need cooperation, $\mu_j = 0$. On the other hand, if a sheet is outside the controllable region, e.g. $d > d_{max}$, the section needs cooperation, $\mu_j = 1$ and cannot help out upstream sections $\varphi_j = 0$. Values in between have been linearly interpolated. Note that $\varphi_j = 1 - \mu_j$ in this case. The expressions for $d_{min} < d < d_s$ are equivalent.

The total control action for a section is a weighted combination of the full cooperation control action, i_{COOP}, and the position control action, i_{ISSC}.

$$\begin{array}{rcl} i_j & \triangleq & (1-\varphi_j)i_{\text{ISSC}} + \varphi_j[(1-\mu_{j-1})i_{\text{ISSC}} + \mu_{j-1}i_{\text{COOP}}] \\ & = & (1-\varphi_j\mu_{j-1})i_{\text{ISSC}} + \varphi_j\mu_{j-1}i_{\text{COOP}} \end{array} \tag{7}$$

Note that this corresponds to some linear interpolation between the two control actions where a hierarchy has been imposed in the sense that i_{ISSC} has priority over i_{COOP}.

One can choose i_{COOP} to be maximum or minimum velocity (1/s-model) in order to help out as much as possible. When a sheet reaches the controllable region boundary, the section switches to maximum or minimum velocity such that the sheet stays within the controllable region. μ is set to one. Cooperation should not compromise on the zero error guarantee at arrival time.

Note that COOP ISSC is defined from the perspective of two neighboring sections, but introduces global cooperation along the paperpath. Indeed, when a section cooperates to reduce an upstream error, the upstream section is in a better position to cooperate with its upstream section. In this way, cooperation travels all across the paperpath and approximates a centralized optimal control scheme with full cooperation among all sections.

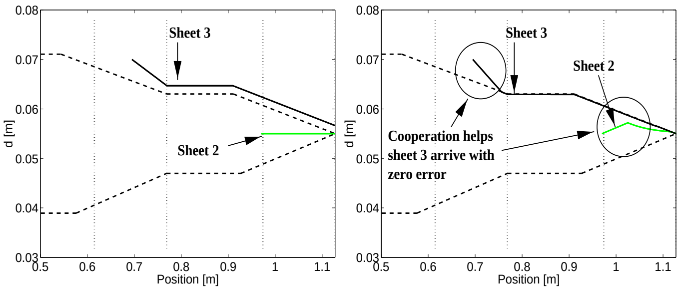


Figure 11: Non-cooperative vs. cooperative ISSC

5.2. COOP ISSC simulation results

Figure 11 illustrates COOP ISSC control for the initial conditions shown in figure 8. The left plot shows the intersheet spacings of sheets 2 and 3 using standard ISSC [2]. Sheet 2 is outside the controllable region and does not arrive in time at the image transfer section. The right plot illustrates COOP ISSC for the same initial conditions. For easy interpretation, φ is chosen 1 as long as a sheet is within its controllable region, 0 otherwise and μ is 1 when outside the controllable region, 0 otherwise.

The controller introduces an error on sheet 2 that helps sheet 3 to catch up. When sheet 3 reaches the controllable region, cooperation stops and the error on sheet 2 is reduced back to 0. Since sheet 3 entered the controllable region during the cooperation, it also arrives at the image transfer section with zero error.

6. Conclusions

This paper presents a non-cooperative, hierarchical hybrid control scheme for a copier paperpath. The control strategy satisfies all the system constraints. How to add cooperation, based on controllable regions, is illustrated for a simplified system model. Simulations illustrate the performance of both strategies.

7. Future Work

COOP ISSC is currently being extended to the $1/s^2$ model. String stability issues could surface when considering disturbances and models beyond $1/s^2$. This mainly depends on the number of sections in the machine.

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